D 11161 **(Pages : 4)** **Name**

Reg. No. . . . .

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Evaluate $\lim_{(x, y, z) \to (1, 0, -1)} \frac{e^{x+z}}{2 + \cos\sqrt{xy}}$.

2. The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the tangent to this parabola at (1, 2, 5).

3. Find the domain and range of the function $f(x, y) = \log_e(y^2 - x^2)$.

4. Find du if $u = \log_e(xyz)$. Find the gradient off $(x, y, z) = xyz$.

5. Find the normal vector to the surface $x^2y - 2y^2z^2 + 17 = 0$ at the point (1, —1, 2).

6. Calculate divergence of $f = x^2\, i + y^2\, j + z2\, \vec{k}$

7. Give the definition of Curl $\vec{f}$

8. Distinguish between flux and circulation of a vector f across/around a plane curve C.

9. What do you **mean by scalar potential ?**

10. **Evaluate** $\int_C 2xy^2\, dS$ **over the unit circle C in the first quadrant.**

11. **Find the maximum value the directional derivative** of $f(x, y, z) = x^2yz^2$ **at (2, 1, —1).**

12. **State Gauss divergence theorem.**

(12 x 1 = 12 marks)

Turn over

Section B

Answer any ten out of fourteen questions.
Each question carries 4 marks.

13. Find $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2}$

14. Find the partial derivatives of z w.r.t x and y respectively at :
(n, π, π) if $\sin(x + y) + \sin(y + z) + \sin(z + x) = 0$.

15. Find $\frac{du}{dt}$ $u = x^3 + y^3$, $x = a \cos t$ and $y = b \sin t$.

16. If $H = f(y\ z, z - x, x - y)$, verify that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \quad = 0$

17. Find the linearization of $f(x, y) = x^2 - xy + y^2/2$ at (3, 2).

18. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2).

19. Find the circulation of the field $\vec{F} = (x-y)\ i + x\vec{j}$ around the circle $x^2 + y^2 = 1$.

20. Evaluate the work done by the force $F = (2x - y + z)\ i + (x + y - z^2)\ j + (3x - 2y\ 4z)\ k$ in moving a particle along the curve C : $r\ (t) = 3\cos t\ \vec{i} + 3\sin t\ \vec{j}$ where $0 < 0$ s 2n.

21. If $r = x\ \vec{i} + y\ \vec{j} + z\ k$, find $\vec{\nabla}\left(\frac{1}{r}\right)$.

22. Find the directional derivative of $f(x, y) = 2x^2 + 3y^2 + z^2$ at (2, 1, 3) in the direction of $\vec{i} - 2\vec{k}$.

23. Find the value of n so that $r^n\ r$ is solenoidal.

24. If a is a constant vector, show that V. $r \times \vec{a} = 0$.

25. If w (X, y, z) has continuous second order partial derivatives, prove that ψ is irrotational.

26. Evaluate $\int_3^4 \int_1^2 \frac{dxdy}{(x+y)^2}$

(10 x 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dxdy$.

28. Change the order of integration and evaluate $\int_0 \int_y e^{x^2}\,dxdy$.

29. Evaluate $\int_C ydx + 2xdy$ where C is boundary of the square in the xy-plane given by $0 \le x \le 1, 0 \le y \le 1$.

30. Show that the differential form $ydx + xdy + 4dz$ is exact and hence evaluate $\int_{(1,1,1)}^{(2,3,-1)} ydx + xdy + 4dz$.

31. If $u = f\left(\frac{y-x}{yx}, \frac{z-x}{zx}\right)$, verify that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.

32. Find Curl Curl f if $f = x^2 y\, i\ 2xz\, j + 2yz\, k$.

33. Compute the local extremum of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$, if any.

34. Using triple integral find the volume of the tetrahedron with vertices (0, 0, 0) ; (1, 1, 0) ; (0, 1, 0) and (1, 1, 1).

Turn over

35. Compute $+ 3z)\, i - (xz + y)\, j + (y^2 + 2z)\, k \,.\, n\, dS$ where S is the surface of the sphere with radius 3 units and centre at (3, 1, -2).

(6 x 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) State Stoke's theorem.

(b) Employing Stokes theorem, evaluate for $\int_C f \,.\, d\, r\; f = (2x - y)\, i - yz^2\, j - y^2 z\, k$ with C as the boundary of the unit sphere in the upper half with centre at the origin.

37. Verify Gauss divergence theorem for f $= x\, i + y\, j + z\, k$ over the sphere $x^2 + y2 + z2\ 1$.

38. (a) Applying the triple integral, find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} =$.

(b) Using double integral find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} \ .1$.

(**2 x 13 = 26** marks)