D 11163 (Pages : 3) Name..............................

Reg. No...........................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Define a countable set.
2. What do you mean by trichotomy law of real numbers ?
3. State Bernoulli's inequality.
4. Find all x satisfying $|x-1|<|x|$.
5. State the completeness property of the set of real numbers.
6. What are the conditions for a subset of real numbers to be an interval ?
7. **If a > 0 find** $\lim \left(\frac{1}{1+na}\right)$
8. State Squeeze theorem for limit of sequences.
9. Give the divergence criteria for a sequence of real numbers.
10. Find *Arg (z) if z=-1*— i.
11. Define contractive sequence.
12. Find the exponential form of $(\sqrt{3}-i$

(12 x 1= 12 marks)

Section *B*

Answer any **ten** *out of* **fourteen** *questions.*
Each question carries **4** *marks.*

13. **Verify that the set of all integers z is denumerable.**
14. **If a 0 and b z 0,** prove that a < 6 if and only if $a^2 < b^2$.
15. **State and prove arithmetic-geometric mean inequality.**

Turn over

16. Define **infimum** of a set. If S = $\{\frac{1}{n} : n \in N\}$, prove that **inf (S) = 0.**

17. If $t > 0$ prove that there is an n_t in N such that $0 < \frac{1}{n_t} < t$.

18. State and prove the **betweenness** property of irrational numbers.

19. Determine the set A of all x satisfying $|2x + 3| < 7$.

20. Test the convergence of the sequence (x_n) if $x_n = \frac{\sin n}{n}$

21. Define Cauchy sequence. Find a sequence (x_n) which is not Cauchy such that $\lim |x_n - x_{n+1}| = 0$.

22. Prove that every convergent sequence of real numbers is a Cauchy sequence.

23. Show that subsequence of a converging real sequence always converge to the same limit.

24. State and prove **Bolzano-Weierstrass** theorem.

25. Find all values of $(-27\,i)^{\frac{1}{3}}$.

26. Prove that $|z_1 - z_2| \geq ||z_1| - |z_2||$ for all $z_1, z_2 \in C$.

(10 x 4 = **40 marks)**

Section C

Answer any **six** *out of* **nine** *questions.*
Each question carries 7 marks.

27. Show that the unit interval [0,1] is uncountable.

28. Prove that there is a real x whose square is 2.

29. If A is any set, prove that there is no **surjection** of A on to the set $\mathcal{P}(A)$ of all subsets of A. Deduce that power set of natural numbers is uncountable.

30. If $I_n = [a_n, b_n]$, $n \in N$ is a nested sequence of closed and bounded intervals, prove that there is a real number which lies in I_n for all n.

31. Sate and prove monotone convergence theorem for a sequence.

32. Show that every contractive sequence is convergent.

33. Discuss the convergence of the following (x_n) where (i) x_n = (1 + 2 ; (ii) x_n $\overline{m!}$.

34. State Cauchy's convergence criterion. Use it to test the convergence of $x_n = \sum_{m=1} m$

35. Find the square roots of $\sqrt{3}$ i and express them in rectangular form.

(6 x 7 = 42 marks)

Section D

Answer any two out of **three** *questions.*
Each question carries **13** *marks.*

36. **(a) State and prove the characterization theorem for intervals.**

(b) Show that between any two real numbers there is a rational number.

37. *(a)* **State and prove the ratio test for the convergence of real sequences.**

(b) If a > 0 construct a sequence of real numbers which will converge to the square root of a.

38. **(a) Let X = (x_n) and Y = (y_n) be real sequences that converge to x and y respectively. Prove the following :**

(i) $(x_n \quad y_n = x + y$.

(ii) $\lim(x_n - y_n) = x - y$.

$\lim(x_n y_n) = xy$.

(iv) $\lim($ $= cx, c \in \mathbb{R}$.

(b) Discuss the convergence of $\frac{n!}{nn}$.

(2 x 13 = 26 marks)