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Name

Reg. No •...............

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

Fill in the blanks :

1. The brachistochrone problem was first solved by ————————

2. Write the heat equation for a rod of finite length completely as a boundary value problem.

3. Find the general solution of $y'' - y = 0$.

4. Find the Laplace transform of $\cosh(2\,at)$.

5. Write the formulas for computing the Fourier coefficients in the Fourier series expansion of a periodic function $f(x)$ of period 2L .

6. Define an exact differential equation. Is $(x + y)dy - \quad - y)dx = 0$ exact ? Why ?

7. Solve the system : $\frac{dx}{dt} = y, \ \frac{dy}{-} = x.$

8. Define unit step function and write its Laplace transform.

9. Give an example of a non-linear differential equation in the dependent variable y and the independent variable x of second order.

10. Show that $u(x, \quad = f(x - ay) + \quad + ay)$ *is* a solution of **the** partial differential equation

$$\frac{\partial^2 u}{\partial x^2} \quad a^2 \frac{\partial^2 u}{dy^2}.$$

Turn over

11. Show that sum and product of two even functions are even functions.

12. Compute the **Wronskian** of the functions et and e^{-t}.

(12 x 1 = **12 marks)**

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries **4** *marks..*

13. (x+ e = 2, y(0)= **0.**

14. **Use Laplace transform to find the solution of** $\frac{d}{d}$ = t, **y (0) = 1.**

15. **Using convolution find the inverse Laplace transform of** $\frac{1}{(s-2)(s-1)}$

16. **Show that any separable equation M** $(x) + N(y)y' = 0$ **is also exact.**

17. **Solve :** $t^2 \quad + ty' + y = 0.$

18. **Use method of variation of parameters to solve :** $y'' + 4y = 3 \operatorname{cosec} t$.

19. **Given that** $y_1 (t) = t$ *is* **a solution of** $2t\ y'' = 3ty - y = 0, t > 0$. **Find a fundamental set of solutions.**

20. **If f (x)** $= x, -\pi \leq x \leq \pi$ **is a** 2π**-periodic function, find** a_n**, the coefficient of** $\cos(nx)$ *in* **its Fourier series expansion.**

21. Find the **values of a and** b **such that the equation** $(ax + by)\frac{dy}{dx} = bx + ay$ *is exact* **and hence solve it.**

22. **Find the Laplace transform of the function :**

$$f(t) = \begin{cases} 2, \text{ if } 0 < x < \pi \\ 0, \text{ if } n < x < 2n \\ \sin t, \text{ if } x > \end{cases}$$

23. State the conditions for the convergence of a Fourier series of a 2π periodic function.

24. Transform the equation u'' + 0.5 u' + u = 0 into a system of first order differential equations.

25. Show that **Wronskian** of the fundamental solutions of y'' + y = 0 is actually non-zero.

26. Write the conditions for the existence of the Laplace transform of a function.

(10 x 4 = 40 marks)

Section C

Answer any six out of nine questions.
Each question carries 7 marks.

27. Solve:

(a) $(3x + 4y)\frac{dy}{dx} = 2x + y$, y(0)=0.

(b) $y - y' = 2xy$, y(0)=1.

28. Find an integrating factor for the equation $(3xy + y^2) + (x^2 + xy)y' = 0$ **and solve it.**

29. Find the general solution of $y'' - 2y' + y = 2\cos(2t) - t^2$.

30. Find the Fourier cosine series expansion of $f(x) = \sin$ (IV **when** $0 < x < L$.

31. Find :

(a) $1(\cosh(at)\cos(at))$.

(b) $\left[\frac{1}{(s^2 + \omega^2)^2} \right.$.

32. **Solve the boundary value problem using Laplace transform :** $y'' - y = 1$, **where** $y(0) = 0, y(\pi/2) = 1$.

33. State and prove Abel's theorem.

Turn over

34. Prove the convolution theorem for Laplace transform.

35. (a) Solve using the method of separation of variables : $\frac{\partial u}{\partial x} = a2 \frac{\partial u}{\partial y}$ (X, o , u(0, = −1

(b) Solve : y" + y' + y = 2t.

(6 x 7 = 42 marks)

Section D

Answer any two out of three questions.
Each question carries 13 marks.

36. Find the Fourier series of :

$$f(x) = \begin{cases} k, \text{ if } \pi/2 < x < \pi/2 \\ 0, \text{ if } \% < x < {}^{3}\% \end{cases}$$

assuming it is period 27r and deduce that $\pi/4 = \mathrm{E}_{n-1} \frac{(-1)^{n+1}}{2n-1}$

37. Find the solution of the initial value problem y" – 2y – 1 = 0, y(0) = 0, y'(0) =1 in two ways ; one of them must be using Laplace transforms.

38. Derive the wave equation by stating the assumptions involved and find its D'Alembert's solution.

(2 x 13 = 26 marks)