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Name..................................

Reg. **No.............................**

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA—I

(2010 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Verify whether the map $(x, y) H (x+1, y)$ is an isometry of the plane which maps the X-axis to X-axis.

2. Verify whether the direct product $Z_2 \times Z_4$ is isomorphic to Z_8.

· Find the order of the element (2, 3) in $Z_3 \times 7Z_6$.

4. For binary codes u, v of same length prove that if $d(u, v) = 0$ then $u = v$.

5. List the elements of the quotient group Z_6 / H where H is the subgroup generated by 2.

6. Give a composition series which is a refinement of (0) s $< 3 >$ 5 Z_{12}.

7. Let H be a subgroup of a group G and G be an H— set defined by $h * g = hg$. Find the orbit of g where g is an element of G and g H.

8. Find the number of sylow 5-subgroups of a group of order 15.

9. If $w = a_1 a_2^2 a3^1$ and $v = a_3 a_2^{-} a_1$ find the reduced word corresponding to wv.

· 10. List all the elements of the group whose presentation is $(x, y : x^{\hat{}} y = xy, xy^{\hat{}} x = y)$.

11. Verify whether $x - 1$ is a factor of $2x - 3x^2 + 1$ in $\mathbf{Q}[x]$.

Turn over

12. Verify whether $x^5 + 3x^3 + 6x + 3$ is irreducible in Q *[x]*.

13. Find the multiplicative inverse of $2i + j + k$ in the skew field of quaternions.

14. Verify whether the ring of all 2 x 2 matrices over ~~is a field.~~

(14 x 1 = **14** weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Prove that $Z_3 \times 7L_5$ is isomorphic to 76_{15}.

16. Prove that if m divides the order of a finite abelian group G then G has a subgroup of order m.

17. Let H be a normal subgroup of a group G and : G G/H be defined by $_{4)}$ *(x)* = *x*H. Show that $\ker\phi = H$.

18. **Let G = H x K be a direct product of groups. Let** $\bar{K} = \{(e, k): k \in K.\}$ **Show that** G/ **isomorphic to H.**

19. **Let X be a G-set. Let** ~ **be a relation on X defined by** x y if y = gx **for some** g e G. Show that - is an equivalence relation on X.

20. **Let H and K be normal subgroups of a group G such that K c H. Show that Gill is isomorphic to** $(G/K)/(H/K)$.

21. **Prove that every group of order 81 is solvable.**

22. **Let** $F[x]$ **be ring of polynomials over a field F. Let** $\phi : F(x) \to F$ **be defined by** $a_0 + a_1x + ... + a_n x^n \mapsto a_0$. **Show that 4) is a homomorphism of rings.**

23. **Show that** $x^3 = 2$ **has no solutions in rational numbers.**

24. **Show that quaternion multiplication is not commutative.**

(7 x 2 = **14** weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage **4.**

25. Define direct product of two groups G_1 and G_2. Prove that if a E G1 is of order m and b E G2 is of order n then order of $(a, b) \in G_1 \times G2$ is the lcm of m and n.

26. Let X be a G-set. For each g E G let $\sigma_g : X \to X$ be defined be $x \mapsto gx$. Prove that $\sigma_{\sim}$ **is** a permutation on X.

27. Define Sylow p-subgroups of a group G. Show that any two Sylow p-subgroups of a group G are conjugates.

28. Show that the set of all quaternions form a skew field, but not a field.

(2 x 4 = 8 weightage)