D 13183 (Pages : 3) **Name..**

Reg. No.....................................

FIRST SEMESTER M.Sc. DEGREE. EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

(2010 Admissions)

Time : Three Hours **Maximum : 36 Weightage**

Part A (Short Answer Type)

Answer **all** *the questions.*
Each question carries weightage 1.

1. **Write an example for a non-trivial subspace of the two dimensional Eu-clidean space ? Justify your claim.**

2. **Define $T : R^2 \to R^2$ by the rule $T(x, y) = (\sin x, y)$. Is T a linear transformation ? Justify your claim.**

3. **Define basis of a vector space.**

4. **Let V be a finite dimensional vector space. What is the minimal polynomial for the identity operator on V.**

5. **Describe explicitly an inner product on R, the set of real numbers.**

6. **Let F be a field and let T be the linear operator on F^2 defined by $T(x_1, x_2) = (x_1 + x_2, x_1)$. Prove that T is non-singular.**

7. **Define hyperspace of a vector space.**

8. **Define inner product space.**

9. **Prove that R^2 is a subspace of the inner product space R^3 with usual inner product.**

10. **Verify whether the vectors (1, 2), (– 2, 1) are orthogonal in R^2.**

11. **Find the characteristic polynomial of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.**

12. **Suppose that $T\alpha = c\alpha$. If f is any polynomial, prove that $f(T)\alpha = f(c)\alpha$.**

13. **State the Cayley-Hamilton theorem.**

14. **Show that an orthogonal set of non-zero vectors is linearly independent.**

(14 x 1 = 14 weightage)

Turn over

Part B (Paragraph Type)

Answer any seven *questions.*
Each question carries weightage 2.

15. Prove that the intersection of any two sub spaces of a vector space V is again a sub space of V.

16. Let [a, *b*] be a closed interval on the real line and C *b*]) be the space of all continuous real valued functions on [a, b]. Then prove that $L(g) = \int_a^b g(t)\,dt$ defines a linear functional on C

17. If W_1 and W_2 are finite dimensional subspaces of a vector space V, then prove that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$.

18. If A is an m x n matrix with entries in the field F, then prove that row rank (A) = column rank (A).

19. Let T be the linear operator on R^2 defined by $T(x_1, x_2) = (x_2, x_1)$. What is the matrix of T in the standard basis for R^2 ?

20. Define annihilator of a subset of a vector space. State and prove the relation that connects the dimensions of a vector space, that of its subspace and of the annihilator space of this subspace.

21. Define transpose of a linear transformation from one vector space to another. If the vector spaces are finite dimensional, prove that rank of a linear transformation is equal to the rank of its transpose.

22. If A and B are n x n complex matrices, show that AB – BA = I is impossible.

23. Define characteristic polynomial of a matrix. Prove that similar matrices have the same characteristic polynomial.

24. Prove that if E is a projection of R along N, then (I – E) is a projection or N along R.

(7 x 2 = 14 weightage)

Part C (Essay Type)

Answer any two questions.
Each question carries weightage 4.

25. Let V be an m-dimensional vector space over the field F and W be an n-dimensional vector spaces over F. Then with usual assumptions prove that the space L (V, W) is a finite-dimensional vector space of dimension mn.

26. Prove that every finite dimensional inner product space has an **orthonormal** basis.

27. Let W be a finite dimensional subspace of an inner product space V and E be an orthogonal projection of V on W. Then prove that E is an **idempotent** linear transformation of V onto W, $W^{\perp}$ is the null space of E, and $V = W$

28. Let W be a finite dimensional subspace of an inner product space V and E be an orthogonal projection of V on W. Then prove that I – E is the orthogonal projection of V on $W^{\perp}$. Also prove that I – E is an **idempotent** linear transformation of V onto $W^{\perp}$ with null space W.

(2 x 4 = 8 **weightage**)