D 13188 **(Pages : 3)** **Name**

Reg. No. . . . .

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 02-LINEAR ALGEBRA

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer all the questions.
***Each question** carries weightage 1.*

1. Let V be a vector space over a field F. Show that if 0 is the scalar zero then 0.a = 0 for all a E V.
2. Verify whether the vector (3, – 1, 0, – 1) is in the subspace of $\mathbf{R}^4$ spanned by the vectors (2, **– 1, 3,** 2), (– **1, 1, 1,** – **3) and (1,** 1, 9, – 5) **?**
3. Let V be a vector space over a field F. Show that if a, 13 and y are linearly independent vectors in V, then a + (3, (3 + y and y + a are linearly independent in V.
4. Let V be the vector space of all n x n matrices over the field F, and let B ε V. If T : V -4 V is defined **by T (A) = AB – BA for A** e V, then verify that T is a linear transformation.
5. Let T be the linear operator on C^3, where C in the field of complex numbers, for which :
   T (1, 0, 0) = (1, 0, i), T (0, 1, 0) = (0, 1, 1), T (0, 0, 1) = (i, 1, 0)
   Is T invertible ? Justify your answer.
6. **Let T be the linear transformation from $\mathbf{R}^3$ into $\mathbf{R}^2$ defined by T (x_1,** x_2, x_3) = ($x_1 + x_2$, $2x_3 - x_1$). If B {(1, 0, - **1), (1, 1, 1), (1, 0, 0)} is an ordered basis for $\mathbf{R}^3$ and B' {(0,1), (1, 0)) is an ordered basis for $\mathbf{R}^2$, then what is the matrix of T relative to $_R$ and**
7. **Show that if W_1 and W_2 are subspaces of a finite-dimensional vector space V, then $W_1 = W_2$ if $W^\circ 1 = W_2$.**
8. **If W is a subspace of a finite-dimensional vector space V and if $\{g_1, g_2, \ldots, g_r\}$ *is* any basis for W°, then show that W = $\bigcap_{i=1} Ng.$,** where $Ng.$ is the null space of g_i.

Turn over

9. Let A be an n x n triangular matrix over the field F. Show that the characteristic values of A are the diagonal entries of A.

10. Let T be the linear operator on R^2, the matrix of which in the standard ordered basis is $A = \begin{bmatrix} 1 & - \\ 2 & 2 \end{bmatrix}$.

Prove that the only subspace of R^2 invariant under T are R^2 and the zero subspace.

11. Let E_1 and $\mathbf{E}_2$ be projections on a vector space V. Show that $E_1 + \mathbf{E}_2 = \mathbf{I}$ iff $E_1 E_2 = 0$.

12. Let (I) be the standard inner product on R^2. Show that for any a in R^2 we have $a = (a / e_1)\, e_1 + (a / e_2)\, e_2$ where $e_1 = (1, 0)$ and $e_2 = (0, 1)$.

13. Let W be a subspace of a finite dimensional inner product space V and E the orthogonal projection of V on W. Show that the mapping $13 \rightarrow R - \mathbf{E}\beta$ is the orthogonal projection of V onto

14. Let V be an inner product space, and let a, R ε V. Show that a = R if and only if $(a / y) = (\beta / y)$ for every y e V.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Show that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S.

16. Suppose P is an n x n invertible matrix over a field F. Let V be an n-dimensional vector space over F, and let B be an ordered basis of V. Show that there is a unique ordered basis B' of V such that

= P [a] B', for every a c V.

17. Let V and W be vector spaces over the same field of dimension n. Show that a linear transformation $T : V \rightarrow W$ is invertible if and only if T is onto.

18. Let V and W be vector spaces over the field F and let u be an isomorphism of V onto W. Prove that T uTu^{-1} *is* an isomorphism of L (V, V) onto L (W, W).

19. Let W be a subspace of a finite-dimensional vector 'space over a field F. Show that :
dim W + **dimW** = dim V.

20. Let V be a finite dimensional vector space over the field F. Show that each basis for $\mathbf{V}^*$ is the dual of some basis for V.

21. Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix

$$A = \begin{matrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{matrix}$$

Show that T is diaonalizable.

22. Show that if V is a finite dimensional vector space and if $V = W_1 \oplus W_2$ ED ... W_K, Then there exists k linear operators $E_1, E_2, \ldots E_k$ on V such that :

(i) $E^2_i = E_i$ for $i = 1,\quad k.$

(ii) $E_i E \quad = 0$, if $i \neq j$.

(iii) $I = \quad + \quad + \ldots + E_k$.

23. Show that if V is an inner product space, then for any vectors α, β in V Ka I 1³)1,- 11a11.IIPII.

24. Let W be the subspace of R^2 spanned by the vector (3, 4), where R^2 is the inner product space with standard inner product. If E in the orthogonal projection of $\mathbf{R^2}$ onto W, find :

(i) A formula for E (x_1, x_2).

(ii) $W^{\perp}$

'(7 x 2 = **14** weightage)

Part C

Answer any two questions.
Each question carries weightage **4.**

25. Let V and W be vector spaces over the **field F and let T be a linear transformation from V into W. Show that if V is finite dimensional then rank (T) + nullity (T) = dim V.**

26. Let W be the subspace of R^5 which is spanned by the vectors $\alpha_1 = (1, 2, 1, 0, 0), \alpha_2 = (0, 2, 3, 3, 1), a_3 = (1, 4, 6, 4, 1)$. Describe W° and find a basis for W.

27. Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Show that T is diagonalizable **if and only if the minimal polynomial for T has the form.**

$p\,(x - c_1)(x - c_2)\ldots(x - c_k)$, **where $c_1, c_2, \ldots c_k$ are distinct elements in F.**

28. Let W be a finite-dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Show that E is an idempotent **linear transformation of V onto W, $W^{\perp}$ is the null space of E, and $V = W\ W^{\mathbf{I}}$.**

(2 x 4 = 8 weightage)