D 13185 (Pages : 4) Name....................................

Reg. No.................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 04—ODE AND CALCULUS OF VARIATIONS

(2010 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer all *questions.*
Each question carries 1 weightage.

1. Define radius of convergence of a power series and determine the same for the power series :

$$\frac{p(p-1)\ \ (p-n+1)\,x}{n!} \quad n=1 \quad ; p \neq 0.$$

2. Locate and classify the singular points on the x-axis for the equation $(3x+1)\,xy'' - (x+1)\,y' + 2y = 0$.

3. Evaluate • $\lim_{b\to\infty} F\left(a, b, a\, \frac{-}{b}\right)$.

4. Determine the nature of the point x ∞ for Legendre's equation $-x^2\ y\ \ 2xy + p(p+1)y - 0$, where p is a constant.

5. Find the first two terms of the Legendre series of $f(x) = ex$.

6. Define gamma function and show that $|(n+\ \ = n!$ for any integer $n > 0$.

7. Show that $\frac{-}{lx}\left[x\, J_1(x)\right] = x\, J_0(x)$.

8. Describe the phase portrait of the system $\frac{dx}{dt} = 1, \frac{dy}{dt} = 2$.

Turn over

9. State Liapunov's theorem.

10. Show that a function of the form a $x^3 + b x^2 y + c xy^{\hat{}} + d y^3$ cannot be either positive definite or negative definite.

11. Find the normal form of Bessel's equation $x^{\hat{}} y + xy' + (x - p^2)$

12. State Picard's theorem.

13. Show that f $y)$ $y^{1/2}$ satisfies a Lipschitz condition on the rectangle I x 151 and $\overset{1}{-}$ $y < 1$.

14. Find the stationary function of $\int_0 \left[xy' - (y)^2\right] dx$ which is determined by the boundary conditions

y (0) = 0 and y (4) = 3.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Show that $\tan x = x + \frac{1}{3} x^3 + \frac{}{5x} + \cdots$ by solving $y' = 1 + y^2$, $y(0) = 0$ in two ways.

16. Find the general solution of :

$(1= e^x) y + \frac{1}{2} y' + e \; y = 0$

near the singular point $x = 0$.

17. Show that the solutions of the equation $(1 \; x^2) y'' - 2xy + n(n+1) y = 0$, where n is a non-negative integer, bounded near $x = 1$ are precisely constant multiples of the polynomial :

$F[-n, n+1, 1, \frac{1}{2}(1-x)]$.

18. If $1(x) = x^p$ for the interval 0 x < 1, show that its Bessel series in the functions $J_\mu(\lambda_n x)$, where

the X, 's are the positive zeros of J_p (~). is $^n = \underset{n=1}{E} \frac{}{\lambda_n J_{+1}(\lambda_n)} J_\mu(\lambda_n x)$.

19. Show that if w (t) is the Wronskian of the two solutions $(x,(t), y, (t))$ and $(x_2\ (t), y_2\ (t))$ on $[a, b]$ of the homogeneous system $\frac{dx}{dt} = \quad (t)\ x + b_1(t)y\ ; \frac{dy}{dt} = a_z(t)x + b_z(t)y$, then w (t) is either identically zero or nowhere zero on $[a, b]$.

20. Determine the nature and stability properties of the critical point **(0, 0)** for the system :

$$\frac{}{dt} = -4x - y, \frac{dy}{dt} = x - 2y.$$

21. Show that (0, 0) is an unstable critical point for the system $\frac{dx}{dt} = 2xy + x^3\ \frac{dy}{dt} = -x^2 + y^5$

22. If y (x) is a non-trivial solution of $y'' + q(x)y = 0$, show that y (x) has an infinite number of positive zeros if $q(x) \frac{k}{x}$ for some $k > \frac{1}{4}$, and only a finite number if $q(x) < \frac{1}{4x2}$.

23. A curve in the first quadrant joins (0, 0) and (1, 0) and has a given area beneath it. Show that the shortest such curve is an arc of a circle.

24. Explain Picard's method of successive approximations of solving the initial value problem :

$y = f(x, y), y(x_0) = y_o$, where $f(x, y)$ is an arbitrary function defined and continuous in some neighbourhoood of the point (x_0, y_0)

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. Find two independent Frobenins series solutions of the equation :

$$x^{\hat{}}y + xy' + (x^2 - 1/4)y = 0.$$

Turn over

26. State and prove the orthogonality property for $J_p(x)$, the Bessel functions of order p.

27. For the non-linear system :

$$\frac{dx}{dt} \quad y\left({}^{2} + 1 \right) \quad \frac{dy}{dt} = 2xy .$$

(i) Find the critical points.

(ii) Find the differential equation of the paths.

(iii) Solve this equation to find the paths.

(iv) Sketch a few of the paths.

28. Solve the initial value problem by Picard's method :

$$\frac{dy}{dx} = z \qquad y(0) = 1$$

$$\frac{dz}{dx} = -y \qquad z(0) = 0.$$

(2 x 4 = 8 weightage)