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Name..............................

Reg. No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 10 03—REAL ANALYSIS--I

(2010 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question has 1 weightage.

1. Define convex set. Give an example of a non-convex set in R_2.

2. For x, y E R^1, let $d(x, y) = |x - 2y|$. Is d a metric on R^1 ? Justify your answer.

3. Is arbitrary intersection of closed sets closed ? Justify your answer.

4. Let E be an infinite subset of a compact set K. Prove that E has a limit point in K.

5. Let f be a real uniformly continuous function on a bounded set E in R_1. Prove that f is bounded on E.

6. Let f and g be differentiable functions on (a, b). Prove that fg is differentiable on (a, b) and $(fg)'(x) = f'(x)\, g(x) + f(x)\, g'(x)$ for all x E (a, b).

7. Evaluate $\lim \frac{\log}{x}$

8. Let a be an increasing function on [a, b], a 5.. x_0 s b, a be a continuous at x_0, $f(x_0) = 1$ and $f(x) = 0$ if x x_0. Prove that f E R (a) on $[a, b]$ and $\int_a^b f\, d\alpha$ O.

9. If f_1 E R (a) and $f_2 \in$ R (a) on $[a, b]$, then prove that $f_1 + f_2$ E R (a) on [a, b] and $\int_a (f_1 + f_2)\, d\alpha$ $\int_a$ $d\alpha + \int_a$ 12 $d\alpha$.

Turn over

10. Let y be a curve in the complex plane, defined on $[0, 2\pi]$ by y $(t) = e^{\cdots}$. Find the length of y.

11. For n = 1, 2, 3,... and for real x, let $f_n(x) = \frac{x}{1+nx^2}$. Show that $\{f_n\}$ is uniformly convergent.

12. Let $\{f_n\}$ be a sequence of real valued differentiable functions that converges to f. Is it true that $f'_n \to f'$? Justify your answer.

13. Define equicontinuity. Prove that functions in an equicontinuous family are continuous.

14. State Stone-Weierstrass theorem.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *from the following ten questions.*
Each question has weightage 2.

15. Prove that every infinite subset of a countable set A is countable.

16. Let X be a set. For $p, q \in X$, define :

$$d(p, q) = \begin{cases} 1 & \text{if } p \neq q \\ 0 & \text{if } p = q \end{cases}$$

Prove that d is a metric on X. Which subsets of the resulting metric space are open.

17. Let E be a closed set of real numbers which is bounded above. If y is the least upper bound of E, then prove that $y \in E$.

18. Let $[x]$ be the largest integer less than or equal to x. What type of discontinuities does the function [x] have ?

19. Let f be a function defined on [a, b]. If f has a local maximum at a point $x \in (a, b)$ and if $f'(x)$ exists, then prove that $f'(x) = 0$.

20. If f is differentiable on [a, b], then prove that f' cannot have any simple discontinuities on $[a, b]$.

21. Let f be a bounded real function on [a, b] and α be monotonically increasing on [a, b]. If P' is a refinement of the partition P, then prove that :

$L(P, f, \alpha) \leq L(P', f, \alpha)$.

22. Let $f \in R(\alpha)$ on $[a, b]$. Prove that $|f| \in R(\alpha)$ on [a, b] and $\left|\int_a^b f\,d\alpha\right| \le \int_a^b |f|\,d\alpha$.

23. Prove that the series :

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$$

converges uniformly in every bounded interval.

24. For n = 1, 2,..., let $f_n(x) = \frac{x^2}{x^2 + (1 - nx)^2}$ Show that $\{f_n\}$ is uniformly bounded on [0, 1]. Also prove that $\{f_n\}$ is not equicontinuous **on [0, 1].**

(7 x 2 = **14** weightage)

Part C

Answer any two from the following four questions.
Each question has weightage **4.**

25. (a) Prove that a set E is open if only if its complement is closed.

(b) Prove that compact subsets of a metric space are closed.

26. State and prove Taylor's theorem.

27. (a) Let f be a bounded function on $[a, b]$. Prove that $f \in R(\alpha)$ if and only if for every $\epsilon > 0$ there exists a partition P of $[a, b]$ such that :

$U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$.

(b) Let f be a bounded function and α be a monotonic increasing function on $[a, b]$. If f_1, f_2 are Riemann-Stieltjes integrable with respect to α on [a, b], then prove that $f_1 + f_2$ is Riemann-Stieltjes integrable with respect to α on $[a, b]$.

28. Let $\{f_n\}$ be a sequence of continuous functions on a set E such that $f_n \to f$ uniformly on E. Prove that f is continuous on E.

(2 x 4 = 8 weightage)