C 4669

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Name..

Reg. No....................................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016

(CUCSS)

Mathematics

MT 2C 07—REAL ANALYSIS—II

(2010 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Short answer questions 1 — 14.
Answer all *questions.*
Each question has 1 weightage.

1. Prove that a linear operator A on a finite dimensional vector space is one-one if and only if the range of A is X.

2. Let 0 be the set of all linear operators on . Let A E ft and B E $L(\mathbb{R}^n)$ with B – A .11 A ' II <1. Prove that B E 11

3. Define gradient of a real valued differentiable function f with domain E, at x a E. Also define the directional derivative of f at x. Illustrate with an example.

4. Prove that the determinant of the matrix of a linear operator on $\mathbb{R}^n$ does not depend on the basis which is used to construct the matrix.

5. If $m^*(A)=0$, prove that $m^*(A\cup B)$ $m^*(B)$.

6. Define Lebesgue measurable sets. Prove that finite sets are measurable.

7. Define measurable functions. Let f be a measurable function and E be a measurable subset of the domain off Prove that f/E is measurable.

8. Define Lebesgue integral of a bounded measurable function. If A and 13 are disjoint measurable sets of finite measure prove that SA,. f = IA f + 5 f.

9. If f and g are bounded measurable functions defined on a set of finite measure, prove that $\int$ (f 4-g)= f + $\int_E g$.

Turn over

10. If f is integrable over a measurable set E, prove that I f I is integrable over E.

11. Give an example of a sequence $\{f_n\}$ that converges in measure but such that $\{f_n(x)\}$ does not converge for any x.

12. Prove that a function f is of bounded variation on $[a, b]$ if and only if f is the difference of two monotone real valued functions on $[a, b]$.

13. Let f be defined by $f(x) = \begin{cases} 0 \text{ if } x=0 \\ x \sin \frac{1}{x} \text{ if } x \quad 0 \end{cases}$. Find $D^+ f(0)$ and $D_+ f(0)$.

14. If f is absolutely continuous prove that f has a derivative almost everywhere.

(14 x 1 = 14 weightage)

Part B

Answer any seven *questions from the following* ten *questions (15 — 24).*
Each question has weightage 2,

15. Let X be an n-dimensional vector space. Prove that every basis of X has n vectors.

16. If 0 is a contraction of a metric space X, prove that 0 has a unique fixed point.

17. Let S be a metric space. Let $a_{11}, a_{12}, \ldots, a_{mn}$ are real continuous functions on S. If for each p E S, A_p is the linear transformation from into le whose matrix has entries $a_{ij}(p)$, prOve that the mapping $p \mapsto A_p$ is a continuous mapping of S into $L(\mathbb{R}^n,$

18. Prove that every borel set is measurable.

19. Let $\{E_n\}$ be an infinite sequence of measurable sets with E_{n+1} E_n for each n. Prove that

$$m\left(\bigcap_{n=1}^{\infty}\right. = \lim_{n \to \infty} m(E_n).$$

20. Let $\{f_n\}$ be a sequence of measurable functions with the same domain of definition. Prove that $\lim f_n$ and $\lim f_n$ are measurable.

21. Let f be a non-negative integrable function. Prove that F defined by $F(x) \int_{-\infty}^{x} f$ is continuous.

22. State and prove the Lebesgue convergence theorem.

23. Let $\{f_n\}$ be a sequence of measurable functions that converge in measure to f. Prove that there is a subsequence $\{f_{n_k}\}$ that converges to f almost everywhere.

24. Show that $T_a^b(cf) = |c|\, T_a^b(f)$ and $T_a^b(f + g) = T_a^b(f) + T_a^b(g)$.

(7 x 2 = 14 weightage)

Part C

Answer any two *questions from the following* four *questions (25 — 28).*
Each question has weightage 4.

25. Let f be a $\mathscr{C}'$ mapping of an open set $E \subset R^n$ into R^n. Let $f'(x)$ is invertible for each $x \in E$. Prove that $f(W)$ is an open subset of R^n for every open set $W \subset E$.

26. Prove that the Lebesgue outer measure of an interval is its length.

27. Prove the Monotone convergence theorem.

28. Let f be an integrable function on [a, b] and $F(x) = F(a) + \int_a^x f(t)\,dt$. Prove that $F'(x) = f(x)$ for almost all $x \in [a, b]$.

(2 x 4 = 8 weightage)