C 4670 (Pages : 3) Name..................................

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016

(CUCSS)

Mathematics

MT 2C 08—TOPOLOGY—I

(2010 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Type Questions)

Answer all *the questions.*
Each question has weightage 1.

1. In a metric space prove that the union of an arbitrary collection of open sets is open.
2. Define discrete and indiscrete topologies on a set X. Give examples for each.
3. Define the Sierpinski topological space.
4. Write an example for a base for the usual topology on the set of real numbers.
5. Define subspace of a topological space. Give an example.
6. Define closure of a subset of a topological space. Prove that if a set is closed, then its closure will be itself.
7. Define accumulation point of a set. Give an example for accumulation point of a set.
8. Define extension problem and lifting problem with regard to continuous functions in topological spaces. How are they related ?
9. Define open map, surjective map and quotient map. State how these three types of maps are related.
10. Define embedding of a topological space into another. Give an example.
11. Is the intersection of any two connected sets connected ? Justify your claim.
12. Define regular and completely regular topological spaces. Write an example for a regular space.
13. State the Tietze characterisation of normality.
14. Prove that regularity is a hereditary property.

(14 x 1 = 14 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **seven** *questions.*
Each question has weightage 2.

15. Prove that the usual topology on the euclidean plane R^2 is strictly weaker than the topology induced on it by the lexicographic ordering.

16. Let $\{x_n\}$ be a sequence in a metric space (X ; *d*). Then prove that {x,,} converges toy in X if and only if for every open *set U* containing y, there exists a positive integer N such that for every integer n N, $x_n \in$ U.

17. If a space is second countable, then prove that every open cover of it has a countable subcover.

18. Let $Z \subset Y \subset X$ and T be a topology on X. Then with usual notations prove that $(T/Y)/Z = T/Z$.

19. Prove that every separable space satisfies the countable chain condition.

20. Prove that a subset A of a space Xis dense in X if and only if for every non-empty open set B of X, A n B *0*.

21. Let C be collection of connected subsets of a space X such that no two members of C are mutually separated. Then prove that $\cup_{C \in C}$ C is connected.

22. Prove that every compact Hausdorff space is T_4.

23. Prove that compactness is weakly hereditary property.

24. Prove that a metric space is a T_3 space.

(7 x 2 = 14 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question has weightage 4.

25. (a) Show that the set of all singleton subsets of a set X is a base for a topology on X.

(b) Define clopen sets in a topology Give an example for a clopen set.

26. (a) Define inerior of a set in a topological space. Let X be a set and A c X. Prove that int(A) is the union of all open sets contained in A. Also prove that it is the largest open subset of X contained in A.

(b) Let X,, X_2 be connected topological spaces and X = X, x X_2 with the product topology. Prove that X is connected.

27. (a) Define Tychnoff space. Prove that every Tychnoff space is T_3.

(b) Suppose y is an accumulation point of a subset A of a T_1 space. The prove that every neighbourhood of y contains infinitely many points of A.

28. (a) Prove that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.

(b) State Urysohn's lemma. Prove the sufficiency condition of the lemma.

(2 x 4 = 8 weightage)