C 4671 **(Pages : 3)** **Name...............................**

Reg. No...............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016

(CUCSS)

Mathematics

MT 2C 09—P.D.E. INTEGRAL EQUATIONS

(2010 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has 1 weightage.

1. Determine a partial differential equation of first order satisfied by the surface F (u, v) = 0, where u = u (x, y, z) and v = v (x, y, z) are known functions of x, y and z and F is an arbitrary function of u and v.

2. Show that $z = ax + (y/a) + b$ is a complete integral of $pq = 1$.

3. Determine the domain in which the two equations $xp - yq - x = 0, x^2 p + q - xy = 0$ are compatible.

4. Find the complete integral of $p + q - pq = 0$.

5. Determine the characteristic curves of the equation $xz_y - yz_x = z$.

6. What are the 'domain of dependence' and the 'range of influence' ?

7. Show that the solution to the Dirichlet problem is stable.

8. State the Cauchy problem for the equation

$$Au. + Bu_{xy} + Cu_{yy} = \mathbf{F}(x, y, u, u_x, u_y),$$

where A, B and C are functions of x and y and give an example.

9. State Heat conduction problem.

10. Determine a suitable Green's function to find the solution of the Dirichlet problem for the upper half plane.

11. Define Voltera equation of second kind and give an example.

Turn over

12. Determine $p(x)$ and $q(x)$ in such a way that the equation $x^2 \frac{d^2y}{dx^2} \quad 2x \frac{dy}{dx} + 2y = 0$ is equivalent to the equation $\frac{d}{dx}\left(p \frac{dy}{dx}\right) + qy = 0$

13. Show that the characteristic functions of the Fredholm equation $y(x) = \int_a K(x, \zeta)\, y \quad d(\zeta)$ corresponding to distinct characteristic numbers are orthogonal over the interval (a, b).

14. Determine the iterated kernel $K_2(x, \zeta)$ associated with $K(x,$ ~~in (0, 1).~~

(14 x 1 = 14 weightage)

Part B

Answer any seven questions.
Each question has 2 weightage.

15. Find the general integral of $(y + 1)p + (x + 1)q = z$.

16. Show that the Pfaffian equation $yz\,dx + (x^2y - zx)\,dy + (x^2z - xy)\,dz = 0$ is integrable and find the corresponding integral.

17. Find the complete integral of $z^2 = pq\,xy$.

18. Solve the Cauchy problem for $2z_x + yz_y = z$, when the initial data curve is C : $x_0 = s, y_0 = s^2, z_0 = s$, $1 < s < 2$.

19. Reduce the equation $x^2u_{xx} - y^2u_{yy} = 0$ into its canonical form.

20. Obtain the D' Alemberts solution which describes the vibrations of an infinite string.

21. Solve the Neumann problem for a circle.

22. Transform the problem $\frac{d^2y}{dx^2} + \quad = x$, $y(0) = 1$, $y'(1) = 0$ to a Fredholm integral equation.

23. Show that the characteristic values of λ for the equation $y(x) = \lambda \int_0^{2\pi} \sin(x + \zeta)\, y(\zeta)\, d\zeta$ are $\lambda_1 = \frac{1}{\pi}$ and $\lambda_2 = -\frac{1}{\pi}$ with corresponding characteristic function of the form $y_1(x) = \sin x + \cos x$ and $y_2(x) = \sin x - \cos x$.

24. Solve the equation by iterative method $y(x) = \lambda \int_0^x (x + \zeta)\, y(\zeta)\, d\zeta + 1$.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question has 4 weightage.

25. Using the method of characteristics, find an integral surface of $p x + q y - z = 0$ containing the initial line $y = 1, x + z = 0$.

26. (a) Solve :

$$y_{tt} = c^2 y_{xx} \qquad 0 < x < 1, t > 0$$

$$y(0, t) = y(1, t) = 0$$

$$y(x, 0) = x(1 - x), \qquad 0 \le x \le 1$$

$$y_t(x, 0) = 0, \qquad 0 < x < 1$$

(b) Show that the solution of the problem in part (a) is unique.

27. Show that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula.

28. Show that any solution of the integral equation $y(x) = \lambda \int (1 - 3x\zeta)\, y(\zeta)\, d\zeta + F(x)$ can be expressed as the sum of F (x) and some linear combination of the characteristic functions.

(2 x 4 = 8 weightage)