C 4672

(Pages : 3)

Name.....

Reg. No.....

# **SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016**

## (CUCSS)

Mathematics

MT 2C 10—NUMBER THEORY

(2010 Admissions)

Time: Three Hours

Maximum: 36 Weightage

## Part A

Answer all questions. Each question has **weightage** of 1.

- 1. If n 1, prove that  $\sum_{din} (d)$  n.
- 2. Define Dirichlet product of two arithmetical functions. If f is an arithmetical function, find I such that f \* I = I \* f = f
- 3. Define a multiplicative function. If f is multiplicative, prove that  $f^{-1}$  is multiplicative.
- 4. If f and g are arithmetical functions, prove that (f \* g)' = f' \* g + f \* g'.
- 5. With usual notations, prove that A \* u = u' and hence derive the **Selberg** identity.
- 6. Prove that [2x] + [2y] [x] + (y] + [x + y].
- 7. Let  $f(x) = x^2 + x + 41$ . Find the smallest integer for which f(x) is composite.
- 8. Solve the congruence  $5x \equiv 3 \pmod{24}$ .
- 9. Find the quadratic residues and non-residues modulo 11.
- 10. State and prove Little Fermat's theorem.
- 11. Determine whether 104 is a quadratic residue or quadratic non-residue of 997.
- 12. Define an affine crypto-system. Illustrate with an example.

**Turn over** 

- <sup>13.</sup> Prove that product of two linear enciphering transformation is a linear enciphering transformation.
- 14. Describe how a signature is sent in RSA.

(14 x 1 = 14 weightage)

#### Part B

Answer any seven questions. Each question has weightage 2.

- 15. If n 1, prove that A (n) =  $(d) \log\left(\frac{n}{d}\right) - (d) \log d.$
- 16. Let  $f_{be a multiplicative function}$ . Show that  $f_{is completely multiplicative if and only if <math>f^{-1}(n) = (n)$  (n) for n = 1.
- 17. State and prove the Euler's summation formula.
- 18. If a and *b* are positive real numbers such that ab = x and land *g* are arithmetical functions with

F (x) = 
$$\bigcup_{n \le x} f(n)$$
 and G (x)  $\sum_{n \ge x} g(n)$ , prove that

$$f(d) g(q) = \mathop{\mathsf{E}}_{n \le a} f(n) \operatorname{\mathsf{G}} + \mathop{\mathsf{E}}_{g(n)} \operatorname{\mathsf{F}}$$
 F(a)  $\operatorname{\mathsf{G}}(b)$ 

- 19. With usual notations, prove that, for x > 0,  $\frac{(\log x)^2}{2\sqrt{x} \log 2} = \psi(x) \vartheta(x)$
- 20. For any prime p 5, prove that  $\sum_{k=1}^{k} \frac{(p-1)!}{k}$  0 (mod p<sup>2</sup>)
- 21. Prove that the Legendre's symbol (n I p) is a completely multiplicative function of n.
- 22. If p and q are distinct odd primes, prove that (p q) (q p) =  $\begin{pmatrix} p 1 \end{pmatrix} \begin{pmatrix} q 1 \end{pmatrix}$

23. In the 27- letter alphabet with 'blank = 26', A =  $\begin{pmatrix} 2 & 0 \\ 7 & 8 \end{pmatrix} \in M2$  (Z/26 Z), to encipher the message

unit "NO" assuming each plaintext message unit  $P = \begin{pmatrix} - \\ y \end{pmatrix}$  is transformed into  $C = \begin{bmatrix} x \\ y \end{pmatrix}$  by the rule

C = A P.

24. Solve the congruences :

 $x + 4y 0 \pmod{9}$ 

 $5x + 8y 0 \pmod{9}$ 

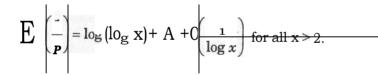
(7 x 2 = 14 weightage)

#### Part C

Answer any two questions. Each question has weightage of 4.

25. Show that  $\frac{1}{6 \log n} \approx \frac{6n}{\log n}$  for every integer n 2.

26. With usual notations, prove that there is a constant A such that



27. (a) Find all odd primes for which 3 is a quadratic residue.

(b) Find all odd primes for which 2 is a quadratic non-residue.

28. Describe algorithm for finding the discrete logs in the finite fields.

 $(2 \times 4 = 8 \text{ weightage})$