D 6717 (Pages : 2) Name..................................

Reg. **No...........................**

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 3C 13—TOPOLOGY—II

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Distinguish between box and large box in product space.
2. Define choice function. Do choice functions necessarily exist ?
3. Define standard sub-base and standard base for the product Topology.
4. Distinguish between locally connectedness and path connectedness.
5. Define the topology of pointwise convergence.
6. Prove that the intersection of any families of boxes is a box.
7. Prove that a subset of X is a box if it is the intersection of a family of walls.
8. Define first Homotopy group of a Topological space.
9. In a simply connected space X, prove that any *two* paths having the same initial and final points are path homotopic.
10. Prove that continuous image of a countably compact space is countably compact.
11. State the embedding lemma in topological spaces.
12. Distinguish between Alexandroff compactification and Stone-Cech compactification of a space X.
13. Define totally bounded subset of a space. Prove that subset of a totally bounded set is totally bounded.
14. Define metrically topologically complete space. Write an example for the same.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question has weightage 2.

15. Justify the term 'box' geometrically for the products of copies of the real line.
16. Let C_i be a closed subset of the space X_i for i E I. Then prove that f a E 1 C_i is a closed subset of

$\prod$ E I X_i with respect to the product topology.

Turn over

17. Prove that the projection functions are open.
18. Prove that a topological product is regular if and only if each coordinate space is regular.
19. Prove that the evaluation function of a family of functions is continuous if and only if each member in the family is continuous.
20. Prove that the fundamental group of unit ball in ${}_n$ is trivial.
21. Prove that a metric space is compact if and only if it is countably compact.
22. Prove that every locally compact, Hausdorff space is regular.
23. Prove that every compact metric space is complete.
24. Prove that a non-empty complete metric space is of second category.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question has weightage 4.

25. Prove that any continuous real-valued function on a closed subset of a normal space can be extended continuously to the whole space.
26. Prove that a subset of X is a large box if and only if it is the intersection of finitely many walls.
27. Prove that metrisability is a countably productive property.
28. Prove that a metric space is compact if and only if it is complete and totally bounded.

(2 x 4 = 8 weightage)