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D 92840 (Pages : 4) Name..

Reg. No.......................................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(CUCBCSS—UG)

Core Course—Mathematics

MAT 1B01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

(Objective Type. Answer all **twelve** *questions)*

1. Find the number of elements in the power set of {letters in the word "yes"}.

2. Find $A \oplus B$ for the sets $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6, 7\}$.

3. What is Anti symmetric relation ?

4. Let $f : A \to B$, when is $f \circ f$ defined.

5. Find the domain of the real valued function $f(x) = x^2 - 3x - 4$.

6. Define a countable set.

7. If the graph of a function is symmetric about the y-axis, then the function is an ————.

8. The graph of $y = x^2$ is shifted 2 units to the right and 2 units up, write the equation for the new graph.

9. At which point is the function $f(x) = \dfrac{x \tan x}{x^2 + 1}$ continuous.

10. Determine whether the statement "If 1 + 1 = 3, then dogs can fly" is True *or* False.

11. Define a Contradiction.

12. Translate the statement "$\exists x \, (C(x) \wedge F(x))$ into English, where C (x) is "x is a comedian" and F (x) is "x is funny" and the domain consists of all people.

(12 × 1 = 12 marks)

Turn over

Part B

(Short Answer Type. Answer any **nine** *questions)*

13. Prove $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

14. Let R be the relation $R = \{(1, a), (2, d), (3, a), (3, b), (3, d)\}$ and $R = \{(b, x), (b, z), (c, y), (d, z)\}$. Find $R \circ S$.

15. Find x and y given $(2x, x + y) = (6, 2)$.

16. Let the function f and g be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find (a) $f \circ g$; and (b) $g \circ f$.

17. Check whether the function defined by $f(x) = 2x - 3$ is bijective.

18. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$ find the number of functions from (a) A into B and (b) B into A.

19. For the function $f(x) = \begin{cases} 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$ find $\lim_{x \to 0} f(x)$ or explain why they do not exist.

20. Evaluate $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$.

21. If $\lim_{x \to 1} h(x) = 5$, $\lim_{x \to 1} p(x) = 1$ and $\lim_{x \to 1} r(x) = 2$ find $\lim_{x \to 1} \frac{\sqrt{5h(x)}}{p(x)(4 - r(x))}$.

22. Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

23. What is the negation of the statement $\exists x (x^2 = 2)$?

24. Translate the statement "Every real number except zero has a multiplicative inverse".

(9 × 2 = 18 marks)

Part C

(Short Essay Type. Answer any **six** *questions)*

25. Find the matrices representation $R \circ S$ where the matrices representing R and S are :

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

26. Suppose ζ is a collection of relations S on a set A and let T be the intersection of relations S, that is $T = \cap \{S \mid S \in \zeta\}$. Prove that if S is symmetric, then T is symmetric.

27. Show that the set Q of rational numbers is denumerable.

28. Let A = {1, 2, 3, 4, 6} and let R be the relation on A defined by "x divides y". Then :

 (a) Write R as a set of ordered pairs.

 (b) Draw its directed graph.

29. Find $\lim_{h \to 0} \dfrac{\sqrt{2+h} - \sqrt{2}}{h}$.

30. State and prove Sandwich theorem for limits.

31. Use logical equivalences to show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

32. Express the statement $\lim_{x \to a} f(x) = L$ using quantifiers.

33. Prove that "If m and n are both perfect squares then nm is also a perfect square".

(6 × 5 = 30 marks)

Part D

(Essay Type. Answer any **two** *questions)*

34. Let A be a set of integers, and let $\sim$ be the relation on A × A defined by $(a, b) \sim (c, d)$ if $a + d = b + c$.

 (a) Prove that $\sim$ is an equivalence relation.

 (b) If A = {1, 2, 3, 4, 5, 6}, find [(2, 5)] the equivalence class of (2, 5).

35. Let $f(x) = \begin{cases} 3 - x & x < 2 \\ 2 & x = 2 \\ \dfrac{x}{2} & x > 2 \end{cases}$

 (a) Find $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2^-} f(x)$ and $f(2)$.

 (b) Does $\lim_{x \to 2} f(x)$ exist ? If so what is it ? If not, why not ?

(c) Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to -1^+} f(x)$.

(d) Does $\lim_{x \to -1} f(x)$ exist ? If so what is it ? If not, why not ?

36. If n is an integer show that the following statement are equivalent :—

$P_1 : n$ is even

$P_2 : n - 1$ is odd

$P_3 : n^2$ is even.

$(2 \times 10 = 20$