

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2011

(C.C.S.S.)

Statistics—Complementary

ST 3C 03_STATISTICAL INFERENCE

(2009 admissions)

Time : Three Hours

Maximum Weightage : 30

Part A

*Answer 11 questions.
Each question carries 1/4 weightage.*

1. The moment generating function of the Chi-square distribution with n.d.f. is :

(a) $(1 - 2t)^{-n/2}$.

(c) $(1 - 2t)^{n/2}$.

(d) None of these.

2. The relation between student's t-distribution and F-distribution is :

(a) $F_{1,1} = t^2$.

(b) $F_{n,1} = t^2$.

(c) $F_{1,1} = t^2$.

(d) None of the above.

3. The S.D. of the sampling distribution of a statistic is known as :

(a) Sampling error.

(b) Standard error.

(c) Means square error.

(d) None of these.

4. Critical region is a region of

(a) Rejection.

(b) Acceptance.

(c) Indecision.

5. By the method of moments one can estimate

(a) All parameters of a population.

(b) Only mean and variance of a distribution.

(c) All moments of a population distribution.

6. If t_1 and t_2 are two estimators such that $\text{Var}(t_1) < \text{Var}(t_2)$ then

(a) t_1 and t_2 equally efficient.(b) t_1 is more efficient than t_2 .(c) t_2 is more efficient than t_1 .

(d) None of these.

Turn over

7. The probability of type II error is •

- (a) Power. (b) 1-power.
(c) Size. (d) None of these.

Let X_1, X_2, \dots, X_n is a random sample from a normal population with mean μ and variance σ^2 and

then the distribution of $\frac{ns^2}{\sigma^2}$ follows Chi-square with $n-1$ degrees of freedom if :

- (a) $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ (b) $S^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$
(c) $S^2 = \frac{1}{n} \sum_{i=1}^n X_i^2$ (d) None of these.

9. Which of the following statement is true ?

- (a) MLE is unbiased. (b) MLE is consistent.
(c) MLE is unique. (d) None of these.

10. Level of significance is the probability of :

- (a) Type I error.
(b) Type II error.
(c) Not committing error.

11. Testing $H_0 : \mu = 10$ Vs. $H_1 : \mu \neq 10$ leads to :

- (a) One-sided upper tailed test.
(b) One-sided lower tailed test.
(c) Two-tailed test.

12. Distribution of the test statistic used to test $H_0 : \sigma^2 = \sigma_0^2$, where σ_0^2 is the variance of a normal population with unknown mean :

- (a) Chi-square distribution with $n-1$ degrees of freedom.
(b) Chi-square distribution with n degrees of freedom.
(c) t distribution with $n-1$ degrees of freedom.
(d) t distribution with n degrees of freedom.

(12 x ¼ = 3 weightage)

Part B

Answer **all** nine questions.
Each question carries 1 weightage.

13. What are the uses of Chi-square distribution ?
14. What do you mean by sampling distribution ?
15. Define an unbiased estimator ?

16. What do you mean by confidence interval ?
17. What are the desirable properties of a good estimate ?
18. Define most powerful critical region.
19. State Fisher Neyman factorization criterion.
20. Write down the application of Neyman Pearson Lemma.
21. Distinguish between Type I and Type II errors.

x 1 = 9 weightage)

Part C

*Answer any five questions.
Each question carries 2 weightage.*

22. State the relation between the normal χ^2 , t and F distribution.
23. Let X_1, X_2, \dots, X_n is a random sample from a normal population with mean μ and variance σ^2 . Obtain a sufficient statistic for σ^2 when μ is unknown.
24. Obtain a 95 % confidence limits for the mean μ of a normal population $N(\mu, \sigma^2)$ when σ^2 is unknown.
25. Give an example of a MLE which is consistent but not unbiased.
26. Distinguish between Simple and Composite hypotheses ? Give examples.
27. Obtain the moment estimator for θ in the population with density given by

$$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta} \quad x \geq 0 \quad \theta > 0.$$

28. Explain the test procedure to test the hypothesis that $H_0: \sigma^2 = \sigma_0^2$ against the alternative $H_1: \sigma^2 > \sigma_0^2$, where σ^2 is the variance of a normal population with unknown mean.

(5 X 2 = 10 weightage)

Part D

*Answer any two questions.
Each question carries 4 weightage.*

29. State and prove additive property of Chi-square distribution.
30. Let X_1, X_2, \dots, X_n is a random sample from a population with density

$$f(x, \alpha) = \frac{e^{-\beta x} x^{\alpha-1}}{\Gamma(\alpha)} \quad 0 < x < \infty,$$

Find the mean and variance of X when α is known. Also find the variance of the mle.

31. Explain the test procedure to test the equality of means of two normal populations with unknown but equal variances.

(2 x 4 = 8 weightage)