Reg. No.....................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2012

(CCSS)

Statistics

ST 3C 03—STATISTICAL INFERENCE

Time : Three Hours Maximum : 30 Weightage

Answer all *twelve* questions.

1. If $(x_1, \ldots, x_n)$ is a r.s. from N $6^2)$ and if $S^2 = \frac{}{n}$, then the statistic $\frac{nS}{2}$ is ————

(a) x^2 -with n *d.f.* (b) x^2 -with n-1 d.f.

(c) t with n (d) t with n-1 d.f.

2. If $x_n)$ is a r.s. from N and if S $= \frac{(x_i}{}$ then V (S^2) ————

(a (b) $\frac{2(n-1)^2}{n}$

(c) $\frac{2(n --}{n}$ (d) 2σ.

If a statistic t is unbiased for a parameter 0, then, which of the following is always true ?

(a) t^2 is unbiased for 0^2 (b) t^2 is not unbiased for 0^2.

(c) t^2 may or may not be unbiased for 0^2 (d) None.

4. Which of the following can be considered as sufficient conditions for consistency

(a) $E(t_n) \to 0, V(t_n)$ 0 as n co. (b) $E(t_n) \to 0, V(t_n) \to 1$ as $n \to 00$.

(c) $E(t_n) \to 0, V(t_n) \to 0$ as $n \to \infty$. (d) $E(t_n \to 0, V(t_n) \to 1$ as $n \to \infty$.

5. The mean and variance of a r.s. of size 100 are 50 and 400 respectively. Then a 95% confidence interval for the population mean is ————

(a) [46.53, 53.47]. (b) [46.43, 53.47].

) [46,25, 53.75]. (d) [46.08, 53.92].

Turn over

6. Let $(x_1, x_2, \quad x_n)$ be a r.s. of size 10 drawn from N(μ, σ Then a 100(1— a)% confidence interval for μ (when σ is unknown) can be constructed using ________ distribution.

(a) Normal. (b) Chi-square.

(c) *t*. (d) F.

7. A 95% confidence interval for the variance of a normal population on a r.s. of size 'n' is given by ________

(a $\left[\dfrac{nS}{\chi^2_{0.025}(n)}, \dfrac{nS}{\chi^2_{0.975}(n)}\right]$. (b) $\left[\dfrac{nS}{\chi^2_{0.025}(n-1)}, \dfrac{nS}{\chi^2_{0.975}(n-1)}\right]$

(c) $\left[\bar{x} \pm t_{0.025} \dfrac{S}{\sqrt{n}}\right]$. (d) None.

8. A 99% confidence interval for the population proportion based on a large sample proportion p is given by ________

(a) $p \pm 2.56 \sqrt{pq}$. (b) $p \pm 1.96 \sqrt{\dfrac{pq}{n}}$

(c) $p \pm 2.56 \sqrt{\dfrac{pq}{n-1}}$ (d) None.

9. The probability of type I error is ________

(a) a. (b) 1 a.

10. The most powerful test consists in minimising ________ or maximising ________ for a fixed ________

(a) a, 13, 1 -P (b) a,1 —a,13.

(c) 1 - ꞵ, a. (d) 13,1 —3, a.

11. For testing H_0 $_0^2$ against the B.C.R. is given by,

$H_1 \cdot \quad > 6_0^2,$

(a) $\dfrac{nS^2}{2} \quad n \; (\alpha)$. (b) $\dfrac{nS^2}{\sigma^2} \; {}^2{}_n\,(a)$.

(c) $\dfrac{nS^2}{6_0} - \chi^2_{n-1}$ (a). $nS \quad {}^2 \quad (\alpha)$.

12. The test statistic for testing the equality of variances of two normal populations based on random samples $(x_1, x_2, \ldots, \quad)$ and $(y_1, y_2, \ldots, y_n)$ *is* ______

(a) $F \quad \frac{x_i \quad {}^{2}\!/_{m}}{(y_i \quad \bar{y}) \quad {}_{n}}$

(b) $\frac{\quad - \bar{\quad}}{S \sqrt{\frac{1}{m} + \frac{1}{n}}}$

(c) $\frac{\Sigma(x_i}{E(Y, -\bar{5})^{2} \quad -1}$

(d) None.

(12 x = 3 weightage)

II. Short answer type questions. Answer all *nine* questions :

13. Define the Chi-square statistic for testing $H_0 \quad \sigma^2 \quad \sigma_0^2$ based on a r.s. $(x1, \quad {}_2, \quad x_n)$ drawn from $N(\mu, \sigma \quad)$ Also, write down the p.d.f. of the statistic.

14. Define an F-variate and write down the p.d.f.

15. Define : 'Consistency' and give a set of sufficient conditions for consistency.

16. Define : (a) Most efficient estimator and (b) Efficiency.-

17. Write down the 100 $(1 - a)\%$ confidence interval for (a) Population proportion and (b) Difference of the means of two normal populations having equal and known variance.

18. Define : (a) Level of significance. and (b) Power of a test.

19. Define : (a) Null hypothesis and (b) Alternative hypothesis.

20. State Neyman-Pearson Fundamental lemma in testing of hypothesis.

21. Write down : (a) The null hypothesis.

(b) Test statistic.

(c) Probability distribution of the test statistic,

associated with the "paired t-test for difference of means". Also, specify the two tailed critical region.

(9 x 1 = 9 weightage)

III. Short Essay *or* Paragraph Questions. Answer any *five* questions

22. Explain the method of maximum likelihood estimation.

Turn over

23. Obtain the moment estimator of the parameter 0 of the exponential distribution,

$$f(x, 0) \quad \begin{cases} \theta e & 0 < x < oo \\ 0 & \text{, elsewhere} \end{cases}$$

24. Let (20, 22, 21, 24, 21) be a r.s. drawn from N , 6 Obtain a 95% confidence interva for υ .

25. If x 0.5 is the C.R. for testing H_0 0 =1 against H_1 = 2, by means of a single observation from a population, $f(x,\theta)=\frac{1}{\theta},0\le x\le$ obtain the sizes of type I and type II errors.

26. Let $(x_1, x_2, \ldots, x_9)$ be a r.s. of size 9 from N(0, 25). If, for testing H_0 = 20, against $H_1 : 0 = 26$, the C.R. is denoted by $W = \{x > 23.266\}$, find the size of the C.R.

27. Show that t-distribution tends to normal distribution under certain conditions.

28. Show that, for an F-distribution with (m, n) d.f., the r raw moment is

$$r = \left(\frac{n}{r}\right) \frac{\left[\frac{}{2}+r\right] \left[\frac{n}{2}-r\right]}{\left[\frac{m}{2}\right] \left[n\right]}$$

what is μ ?

(5 x 2 = 10 weightage)

IV. Essay type questions. Answer any *two* questions.

29. (a) Show that if F has an F-distribution with (, n_2) d.f., — has an F-distribution with (n_2, n_1) di

(b) Also, explain the relation between *t* and F distributions.

30. (a) Find the moment estimator and m.l.e. for the parameter 0 if the distribution, $f(x\ 0) = (0+1)$, $0\ x$ 1.

(b) If a r.s. of size 8 from this population produces the data :

0.2, 0.4, 0.8, 0.5, 0.7, 0.9. 0.8, 0.9,.

find that values of moment estimator and m.l.e. of 0 .

31. Explain (i) the large sample test for the equality of population proportions ; and (ii) chi-square test for the independence of two attributes.

(2 x 4 = 8 weightage)