

# THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(U.G.—CCSS)

(Complementary Course)

MM 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

## I. Objective Type Questions : Answer *all* questions :

Each question of weightage

1 Check for exactness :

$$(x^3 + 3xy^2) dx + (3x^2y + y^3) dy = 0.$$

2 Curves that intersect a given curve at right angles are called \_\_\_\_\_

3 Solve  $y' = \log x$ .

4 When is a square matrix A said to be non-singular ?

5 Find the characteristic roots of  $\begin{bmatrix} 1 & 3 \\ 10 & 21 \end{bmatrix}$

6 If '2' is an eigenvalue of a square matrix A, give one root of AT.

7 Define a solenoidal vector.

8 When are two vectors  $\vec{a}$  and  $\vec{b}$  said to be orthogonal ?

9 What is the divergence of  $\vec{a} = [x^2, y^2, z^2]$  ?

10 Find a unit normal vector to the surface  $S : x^2 + y^2 + z^2 = a^2$ .

11 Give the parametric representation of the plane  $3x + 2y + z = 6$ .

12 If  $\vec{F} = \text{grad } f, f = x^2 + y^2 + 2z^2$ , find  $\int_C \vec{F} \cdot d\vec{r}$  where C has initial point A (0, 0, 0) and terminal point B : (2, 2, 2).

(12 X ¼ = 3 weightage)

II. Short Answer Type Questions : Answer *all* questions.

Each question of weightage 1.

13 Find the rank of the following matrix

$$A = \begin{vmatrix} 4 & 7 & 6 \\ 8 & 14 & 12 \\ 11 & 1 & 1 \end{vmatrix}$$

14 State Cayley Hamilton Theorem.

15 Solve  $(1+x^2) \frac{dy}{dx} = 1+y^2$ ,  $y(0)=1$ .

16 Find an integrating factor for  $2x \tan y \, dx + \sec^2 y \, dy = 0$ .

17 Find the work done by  $\vec{p} = [2, 6, 6]$  if it displaces a body from A : (3, 4, 0) to B : (5, 8, 0)

18 Prove that  $\text{div}[\text{grad } f] = \nabla^2 f$

19 Find the tangential and normal accelerations of  $\vec{r}(t) = [b \cos t, b \sin t, c]$ .

20 Check for path independence :

$$\sinh xz (zdx - xzdz).$$

21 Use Green's Theorem to evaluate  $\oint_C \vec{F} \cdot d\vec{r}$ , where  $\vec{F} = \text{grad} (\sin x \cos y)$  and C is the ellipse

$$25x^2 + 9y^2 = 225.$$

(9 x 1 = 9 weightage)

III. Short Essay or Paragraph Questions : Answer any *five* questions :

Each question of weightage 2.

22 Solve  $2xyy' = y^2 - x^2$ .

23 Find the rank by reducing to normal form :

$$A = \begin{vmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{vmatrix}$$

24 Find the eigenvalues of  $A = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix}$

25 Find the directional derivative of  $f(x^2 + y^2 + z^2)^{1/2}$  at  $(3, 0, 4)$  along  $\vec{a} = [1, 1, 1]$ .

26 Find the length of the hypocycloid  $(t) = a \cos^3 t \mathbf{i} + a \sin^3 t \mathbf{j}$ .

27 Test for exactness and hence evaluate :

$$\int_{(4, 1/2)} 2x \sin \pi y \, dx + \pi x^2 \cos \pi y \, dy.$$

28 Evaluate using the Gauss Divergence Theorem :  $\vec{F} = [x^3, y^3, z^3]$  and S is the surface of the sphere  $x^2 + y^2 + z^2 = 9$ .

(5 x 2 = 10 weightage)

IV Essay Questions : Answer any *two* questions.

Each question of weightage 4.

29 Find the Orthogonal Trajectories of  $y = c\sqrt{x}$

30 Use Cayley Hamilton Theorem to find  $A^3$  and  $A^4$  if  $A = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix}$ .

31 Verify Stokes' Theorem for  $\vec{F} = [y, z, x]$  and S is the portion of the paraboloid  $z = 1 - x^2 - y^2$ ,  $z \geq 0$ .

(2 x 4 = 8 weightage)