

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2012

Mathematics—Core Course

MM 3B 03—CALCULUS

Time : Three Hours

Maximum : 30 Weightage

Part I

I. Answer all the *twelve* questions1 Write the domain and range of the function $y = \sqrt{1 - x^2}$ 2 If $f(x) = x + 5$, $g(x) = x^2 - 3$ find $(g \circ f)(0)$.3 Graph the function $f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$ 4 What real numbers satisfy the equation $[x] = 0$?5 Find $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^3 - 4x}$ 6 For what value of 'a' is $f(x) = \begin{cases} x - 1, & x \leq 3 \\ ax + 3, & x > 3 \end{cases}$ continuous for every x?7 Find the critical points of the function $f(x) = x^4 - 4x^3 + 10$.8 Define linearization of $f(x)$ at $x = a$.9 Express $1 - 2 + 4 - 8 + 16 - 32$ in sigma notation.10 Find the average value of $f(x) = x^2 - 1$ is $[0, \sqrt{3}]$.11 Find the work done by the force $f(x) = 30x$ along x axis from $x = 0$ m to $x = 2$ m,

12 Write the shell formula for revolution about y axis.

(12 x $\frac{1}{4}$ = 3 weightage)

Part II

II. Answer all the *nine* questions

13 Find the vertex and axis of the parabola $y = -\frac{1}{2}x^2 - x + 4$.

14 $f(x) = \frac{x+1}{x-1}$. Does f^{-1} exist? Why?

15 At what points does the function $y = \ln x + \sin x$ is continuous?

16 Find the value of c in the mean value theorem for the function $f(x) = x + \frac{1}{2}$ in $\left[\frac{1}{2}, 1\right]$.

17 Use Sandwich theorem to find the asymptotes of the curve $y = 2 + \sin x$.

18 Evaluate $\int_0^4 3x \sqrt{x^2 - 1} dx$.

19 Find the area between $y = \sec x$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{4}$.

20 Define moment of a system about origin.

21 Write the formula for finding centre of mass of a thin rod along x axis with density $\delta(x)$.

(9 x 1 = 9 weightage)

Part III

III. Answer any *five* questions from seven:

22 $f(x) = \frac{(x+3)(x+2)}{x+2}$ find $f^{-1}(x)$ and $f(f^{-1}(x))$

23 Test the continuity of the function $f(x) = x \sin \frac{1}{x}$

24 Find two positive integers whose sum is 20 and whose product is as large as possible.

25 Find the area of the region between and the graph of $f(x) = x^3 - x^2 - 2x$, $1 < x < 2$.

26 Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about line $x = 3$.

2 Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/4} - 1$ for $0 < x < 1$

28 Find the centre of mass of a thin plate of constant density and covering the region bounded by the parabola $y = x - x^2$ and the line $y + x = 0$.

(5 x 2 = 10 weightage)

Part IV

Answer any *two* questions from three :-

29 $\lim_{x \rightarrow 5} \sqrt{x} - 1 = 2$. Find a $\delta > 0$ that works for ϵ

30 Find the asymptotes of the curve

$$f(x) = \frac{x^3}{2x - 1} \text{ and find the dominant terms.}$$

31 Find the area of the surface generated by revolving the curve $y = \frac{x^3}{9}$, $0 \leq x \leq 1.2$ about x axis.

(2 x 4 = 8 weightage)