Name ....

Reg. No. ....

THIRD SEMESTER B.Sc. (COMPUTER SCIENCE) DEGREE EXAMINATION NOVEMBER 2010

(CCSS)

Statistics—Complementary Course

ST3 C03—STATISTICAL INFERENCE

Time : Three Hours Maximum Weightage : 30

I. Answer all *twelve* questions

1 If , $x_2, \ldots,$) is a r.s. from N(μ, a^2 , then the statistic $\bar{S/}$ - 0- is ———

(a) t-variate with n d.f. (b) t-variate with n – 1 d.f.

(c) x^2 -variate with n d.f. (d) x^2 -variate with n – 1 d.f.

2 The square of a t-variate with n d.f. is ———

(a) a t -variate with n d.f. (b) a t - variate with n^2 d.f.

(c) F-variate with (1, n) d.f. (d) F-variate with (n, 1) d.f.

3 The theory of estimation was founded by :

(a) R.A. Fisher. (b) Neyman.

(c) Pearson. (d) C.R. Rao.

4 Let ($x_1, x_2, \ldots, x_n$) be a r.s. from N(μ, σ Then, which of the following estimators is unbiased for σ ?

$(x_i - \bar{x})$

$n - 1$

(c) $\sum(x_i - \bar{x})$ ——— (d) $\sum(x_i$ 2

n

5 The mean of a r.s. of size 100 from N μ, 6^2) is 30. If a^2 = 25, then a 95% confidence interval for p is :

(a) [29.02, 30.98]. (b) [27.5, 32.5].

(c) [25.1, 34.9]. (d) None.

6 Let $x_1, x_2, \ldots x_n$ be a r.s. from N(μ, a^2 . Then a 100 (1 – $\propto$) % confidence interval for a^2 can be constructed using ——— distribution.

(a) Normal. (b) Chi-square.

(c) t. (d) F.

Turn over

7 A 100 (1 $\propto$)% confidence interval for a^2 of N(σ , based on a r.s. of size n is given by :

(a) $\dfrac{ns}{\chi^{2(n-1)}_{\propto/2}}$ $\dfrac{ns}{2(n-1)_{-\propto/2}}$

(b) $\left|\bar{X} - t_{\propto/2} \cdot \frac{s}{\sqrt{n}}\ \mathbf{x} + t_{\propto/2} \cdot \frac{}{\sqrt{n}}\right|$

(c) $\left[\bar{X} - z_{\frac{\propto}{2}} \cdot \cdot \frac{s}{\sqrt{n}}, \bar{x} + z_{\frac{\propto}{2}} \cdot \frac{s}{\sqrt{n}}\right]$.

(d) None.

8 A 95% confidence interval for the population proportion based on a large sample proportion p is given by ________

(a) $p \pm 1.95 \sqrt{\dfrac{pq}{n-1}}$

(b) P $1.95 \sqrt{\frac{pq}{}}$.

(c) $p \pm 1.96$

(d) $p \pm 1.96$ pq

9 The theory of testing of hypothesis was initiated by ________

(a) A.Wald.

(b) R.A. Fisher.

(c) C.R. Rao.

(d) Neyman and Pearson.,

10 The Neyman-Pearson lemma provides the B.C.R. for testing ________ hypothesis against ____ alternative.

(a) Simple, simple.

(b) Simple, composite.

(c) Composite, simple.

(d) Composite, composite.

11 For testing :

$H_0 : \mu = \mu_0$ against

H_1 $< \mu_0$, based

on a large sample, the B.C.R. is given by :

(a) $\dfrac{-\mu_0}{s/\sqrt{n}} < Z$

(b) $\dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$ $Z_\propto$.

(c) $\bar{X}$ __ μ_0 $<$ $Z_\propto$.

(d) None.

where $Z_\propto$ is such that $P\{Z > Z_\propto = \propto$

12 The test for equality of variances of two normal populations is based on ________ distribution.

(a) Normal.

(b) t.

(c) x^2.

(d) F.

(12 x V4 = 3 weightage)

III. Short answer type questions. Answer all *nine* questions :

13 Define (i) Statistic ; (ii) Sampling distribution. Given an example for each.

14 (i) Define a t-statistic for testing : H_o : $= \mu_0$ based on a r.s. from N(μ, σ .

(ii) Also write down the p.d.f. of the statistic.

15 (i) Define unbiasedness and consistency.

(ii) Give an example for an estimator which is unbiased as well as consistent.

16 Define 'Sufficiency' and state the Neyman factorization theorem.

17 Define : (i) Confidence interval;

(ii) Confidence coefficients.

18 Define : (i) Type **I** error; and

(ii) Type **II** error.

19 Define : (i) Simple hypothesis; and

(ii) Composite hypothesis.

Give an example for each.

20 Define : (i) Critical Region; and

(ii) Most powerful critical Region.

21 Write down the test statistic and Best critical Region associated with the test for goodness of fit.

(9 x 1 = 9 weightage)

III. Short Essay *or* paragraph questions. Answer any *five* questions :

22 Obtain the m.g.f. of Chi-square distribution and show that the distribution satisfies additive property.

23 (i) Obtain the mode of F-distribution.

(ii) What is the mode of the F-distribution with (4, 3) d.f. ?

24 Explain the method of moments.

25 Obtian the moment estimator of the parameter X of the Poisson distribution P(λ).

26 The sample variance $S^2 = \dfrac{E\,(x_i - \bar{x})^2}{n}$, based on a r.s. of 15 observations, from a Normal population is 12. Obtian a 95% confidence interval for the population variance.

Turn over

27 If $x > 1$ is the CR for testing $H_o : O = 2$ against the alternative $H_1 : O = 1$, on the basis of a single observation from the population, $f(x, O) : \theta e^{-x}$ $0 < x < \infty$. Obtain the probability of type I and type II errors.

28 Find the power of the test for testing $H_o : O = 2$ against $H_1 : O = 1$ based on a r.s. (x_1, x_2) of size 2 from a population, $f(x, 0) = \frac{-}{0} e^{-x/\theta}$, $x < CO$.

Take the C.R. as $C = \{(x_1, x_2) : 9.5 \leq x_1 + x_2)$.

(5 x 2 = 10 weightage)

IV. Essay type questions. Answer any *two* questions :

29 (i) Define t-distribution and show that $\mu_{2r} = \frac{n^r (2r-1)(2r-3)\ldots 3.1}{(n \quad 2)(n \quad 4)\cdots(n \quad 2r)} \frac{n}{2} \; r$ and $\mu_{2r+1} = 0$, for $r = 1, 2, 3, \ldots$

(ii) Also, obtian the and y coefficients and interpret the Skewness and Kurtosis.

30 In random sampling from normal population $N(\;, 6^2)$ find the maximum likelihood estimators for :

(i) μ when $\sigma^{\hat{}}$ is known

(ii) $\sigma^{\hat{}}$ when μ is known, and

(iii) simultaneous estimation of μ and a^2.

31 Explain (i) the t-test for equality of means and ; (ii) F-test for equality of variances of two normal populations.

(2 x 4 = 8 weightage)