

THIRD SEMESTER B.Sc. DEGREE EXAMINATION NOVEMBER 2014

(U.G.-CCSS)

Complementary Course

MM 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Section A

*Answer all questions.**Each question carries $\frac{1}{4}$ weightage.*

1. Show with an example that scalar product of vectors is commutative.
2. Find the acceleration of a particle with position vector $\vec{r}(t) = [\sin t, 0, 0]$.
3. If $f = x^2 + y^2 + z^2$, find $\text{grad } f$
4. What is the Cartesian form of $\vec{r}(u, v) = [u \cos v, u \sin v, u]$?
5. If $\mathbf{F} = \text{grad } f$, then $\text{curl } \vec{\mathbf{F}}$ _____
6. Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 9$
7. Verify that $y = e^x + ax^2 + bx + c$ is a solution $y' = e^x$
8. Solve $y' = -2xy$
9. Test for exactness : $-\frac{y}{x^2} dx - \frac{x}{y^2} dy = 0$.
10. Define rank of a matrix.
11. Is the matrix $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ singular or non-singular?
12. State Cayley Hamilton theorem.

(12 x $\frac{1}{4}$ = 3 weightage)

Turn over

Section B

Answer all questions.

Each question carries 1 weightage.

13. Find the angle between $x + y + z = 1$ and $x + 2y + 3z = 6$.
14. Find a parametric representation of the straight line through $(4, 2, 0)$ in the direction of $[1, 1, 0]$.
15. Find the length of the semi cubical parabola $\vec{r}(t) = t, t^{3/2}, 0$ from $(0, 0, 0)$ to $(4, 8, 0)$.
16. Evaluate $\int_C \text{grad } r, F(\vec{r}) = [z, x, y], C : F(t) = [\cos t, \sin t, 3t]$ and $0 < t \leq 2\pi$.
17. Use Green's theorem to find the area enclosed by the circle $x^2 + y^2 = a^2$.
18. Solve the initial value problem $2\sin 2x \sinh y dx - \cos 2x \cosh y dy = 0, y(0) = 1$.
19. Find an integrating factor : $2 \cosh x \cos y dx = \sinh x \sin y dy$
20. Find the rank of $\begin{vmatrix} 1 & 3 & 6 \\ 2 & 6 & 12 \end{vmatrix}$
21. Find the eigen values of $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$.

(9 x 1 = 9 weightage)

Section C

Answer any five questions.

Each question carries 2 weightage.

22. (i) Find the potential function of $[yz, xz, xy]$.
 (ii) Test whether irrotational $\vec{F} = [2y^2, 0, 0]$.
23. Test for path independence and if independent, integrate from $(0, 0, 0)$ to $(a, b, c) : \cos(x + yz) [dx + zdy + ydz]$.
24. Evaluate $\oint_S \vec{F} \cdot \vec{n} dA$ using Gauss divergence theorem
 $\vec{F} = x^3, y^3, z^3$, S is the surface of the sphere $x^2 + y^2 + z^2 = 4$.

25. Solve $y' + y \sin x = e^{\cos x}$

26. Solve using the transformation $v = xy : 2xyy' = y^2 - x^2$.

27. Find the rank by reducing to normal form : $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$

28. Using Cayley Hamilton theorem. Find the inverse of : $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

(5 x 2 = 10 weightage)

Section D

Answer any two questions.

Each question carries 4 weightage.

29. State Stokes' theorem and verify it for $S = [y^2, z^2, x^2]$ S being the portion of the paraboloid

$$x^2 + y^2 = z, y$$

30. (i) Solve $y' + 2y = y^2$.

(ii) Find the Orthogonal trajectories of $y = ce^x$.

31. Find the eigen values and eigen vectors of

$$A = \begin{bmatrix} -2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{bmatrix}$$

(2 x 4 = 8 weightage)