

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(UG-CCSS)

Core Course—Mathematics

MM 3B 03—CALCULUS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* twelve questions :

1 Evaluate $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}$.

2 State the Sandwich theorem.

3 Define the intermediate value theorem.

4 At what points are the function $Y = \frac{\cos x}{x}$ is continuous.

5 State the first derivative theorem for local extreme values.

6 Define the critical point of a function f .7 What are the critical points of f given $f'(x) = x(x-1)$?8 Find the intervals in which the function f is increasing given $f'(x) = x(x-1)$.

9 Evaluate $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

10 Find dy if $y = x^5 + 37x$.11 Write the sum without sigma notation and then evaluate the sum $\sum_{k=1}^2 6k$.12 Suppose that $\int_1^2 f(x) dx = 5$. Find $\int_1^2 f(u) du$.(12 x $\frac{1}{4}$ = 3 weightage)II. Short Answer Type Questions. Answer *all* nine questions :13 Find the work done by a force of $F(x) = \frac{1}{2} N$ along the x-axis from $x = 1$ m to $x = 10$ m.14 A spring has a natural length of 1 m. A force of 24 N stretches the spring to a length of 1.8 m. Find the force constant k .

Turn over

15 Find the volume of the solid generated by revolving the region between the y-axis and the

curve $x = \frac{2}{1} x^4$ about the y-axis.

16 Evaluate $\int_{-1}^1 3x^2 \sqrt{x^3 + 1} dx$

17 Find $\frac{dy}{dx} \cdot y \int_1^{x^2} \cos t dt$.

18 Find the average value of $f(x) = 4 - x^2$ on $[0, 3]$.

19 Show that the value of $\int_0^1 \frac{1}{1 + \cos x} dx$ cannot possibly be 2.

20 Evaluate $\sum_{k=1}^4 (k^2 - 3k)$.

21 Find the linearization of $f(x) = \cos x$ at $x = \frac{\pi}{2}$

(9 x 1 = 9 weightage)

III. Short Essay or Paragraph questions. Answer any *five* questions

22 Show that the centre of mass of a straight thin strip or rod of constant density δ lies halfway between its two ends.

23 Find the lateral surface area of the cone generated by revolving the line segment $y = \frac{x}{2}, 0 \leq x \leq 4$ about the x-axis.

24 Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

25 Evaluate $\int_0^{\pi/6} \cos^{-1} 20 \sin 20 d\theta$.

26 Find the total area between the region $y = -x^2 - 2x, -3 < x < 2$ and the x-axis.

27 Express the solution of the following initial value problem as an integral

Differential equation: $\frac{dy}{dx} = \tan x$

Initial condition : $y(1) = 5$.

28 Show that among all rectangles with a given perimeter the one with the larger area is a square.

(5 x 2 = 10 weightage)

IV. Essay questions. Answer *two* questions :

29 A 10 m. long rod with thickness 5.52 has density $\delta(x) = 1 + \frac{x}{10}$ kg/m. Find the rod's centre of mass.

30 Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

31 The cost function at American Gadget $C(x) = x^3 - 6x^2 + 15x$ (x is thousands of units). Is there a production level that minimize average cost ? **If so, what is it ?**

(2 x 4 = 8 weightage)