

**SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT)
EXAMINATION, APRIL/MAY 2015**

(UG—CCSS)

Complementary Course—Mathematics

MM 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

I. Objective Type Questions : (Answer *all* questions.)

1 Differentiate $\coth 5x$.

2 Show that $\sinh 2x = 2\sinh x \cosh x$.

3 Integrate $\operatorname{sech}^{-1}\left(x - \frac{1}{2}\right)$.

4 Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

5 Give an example of a constant sequence.

6 $\lim_{n \rightarrow \infty} \sqrt[n]{n} =$ _____

7 State Alternating Series Test (Leibniz's Theorem).

8 Write the series for $\ln(1+x)$.

9 Find the Taylor polynomial of order 0 generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$.

10 $|r|=1$ is the equation for a _____

Find $\frac{dy}{dx}$ if $x^2 + \sin y - 2y = 0$.

12 Define gradient vector.

(12 x ¼ = 3 weightage)

Turn over

II. Short Answer Type Questions : (Answer all *nine* questions).

13 Find f_x if $f(x, y) = \frac{2y}{y + \cos x}$

14 Find an equation for the hyperbola with eccentricity $\frac{1}{2}$ and directrix $x = 2$.

15 Graph the set of points whose polar co-ordinates satisfy the conditions $-3 \leq r \leq 2$ and $0 \leq \theta \leq \frac{\pi}{4}$

16 Find the Maclaurin's series for $f(x) = \frac{1}{1+x}$

17 For what values of x do the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges absolutely.

18 Given $a_1 = a_2 = 1$, $a_{n+2} = a_{n+1} + a_n$. Write the first 4 terms of the sequence.

19 Investigate the convergence of $\sum_{n=1}^{\infty} \frac{dx}{(x-1)^{n+1}}$.

20 Evaluate $\int_0^1 \cosh(\ln t) dt$.

21 Use the definition of $\cosh x$ and $\sinh x$ to show that $\cosh^2 x - \sinh^2 x = 1$.
(9 x 1 = 9 weightage)

III. Short Essay Questions. (Answer any *five* questions).

22 Is the area under the curve $Y = \frac{\ln x}{x^2}$ from $x = 1$ to $x = \infty$ finite. If so, what is it?

23 Investigate the convergence of the series

24 Find the Taylor series generated by :

$$f(x) = x^4 + x^2 + \text{ at a } -2.$$

25 Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.

26 Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is continuous at every point except the origin.

27 Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s $w = x^2 + y^2, x = r - s, y = r + s$.

28 Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$

(a) Increases most rapidly ; and

(b) Decreases most rapidly at the point (1, 1).

(5 x 2 = 10 weightage)

IV. Essay Questions. (Answer any two questions)

29 Find the linearization $L(x, y, z)$ of $f(x, y, z) = x^2 - xy + z \sin z$ at the point (2, 1, 0). Find an upper bound for the error occurred in replacing f by L over the rectangle $R: |x - 2| \leq 0.01, |y - 1| \leq 0.02, |z| \leq 0.01$.

30 Find the length of the cardioid $r = 1 - \cos \theta$.

31 Evaluate $\int_2^{\infty} \frac{x+3}{(x^2+1)^2} dx$.

(2 x 4 = 8 weightage)