Reg. No.................................

SECOND YEAR B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2009

Part III Mathematics (Main)

Paper II DIFFERENTIAL AND INTEGRAL CALCULUS

(2001 admission onwards)

Time : Three Hours **Maximum : 60 Marks**

Maximum marks that can be earned from Unit I is 15, *Unit II is* 15, *Unit III is* 10 *and Unit IV is* 20.

Unit I

(Maximum 15 *marks can be awarded)*

1. Show that the radius of curvature at the point 0 of the curve x = 3a cos° – a cos 30, y = 3a sine – a sin 30 is 3a sine.

(5 marks)

2. Show that the evolute of $\frac{x^2}{a} + \frac{y^2}{b^2} = 1$ is $ax^{2/3} + (by)^{2/3} = (a2\ b2)^{2/3}$ (5 marks)

3. Find all the asymptotes of $(x + y)^2 (x + 2y + 2) – (x + 9y – 2) = 0$. (5 marks)

4. Locate the double point on the curve $y (y – 6) = x^{-} (x – 2)^3 – 9$. (5 marks)

5. Trace the curve $a^{-} y^2 = x2 (a2 \_x2)$. (5 marks)

Unit II

Maximum 15 *marks can be awarded.*

6. Find the reduction formula for $\int \sin\ x\, dx$. (4 marks)

7. Evaluate $\int_0^{\pi/2} \sin^{-} x \cos^y x\, dx$. (4 marks)

8. Prove that the area of the loop of the curve $y^2 (a + x) = x^2 (a\ x)$ is $2a^2 \left(1 — \frac{-}{4}\right)$. (4 marks)

9. The area bounded by the curve $x = a \cos^{-}\theta$, y = a $\sin^{-}$ 0 and lying in the first quadrant revolves about the x-axis. Find the volume of the solid generated.

(4 marks)

10. Find the curved surface generated by the revolution about the x-axis of the portion of the parabola $y^2 = 4ax$ included between the origin and the ordinate x = 3a.

(4 marks)

11. Find the moment of inertia of a solid sphere about an axis passing through its centre. (5 marks)

Turn over

Unit III

Maximum **10** *marks can be awarded*

12. Using Maclaurin's theorem expand log(1+ e^x . **(4 marks)**

13. Show that $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\cdots=1-\log 2.$ **(4 marks)**

14. Find the sum to infinity of the series $1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\cdots.$ **(4 marks)**

15. Sum the series $\frac{1^2}{1!}+\frac{1^2+2^2}{2!}+\frac{1^2+2^2+3^2}{3!}+\cdots.$ **(5 marks**

Unit IV

Maximum* 20 *marks can be awarded

16. Show that $f(x,y)=\frac{2xy}{x^2+y^2}$ $(x,y)\neq(0,0)$ **is continuous at every point except the origin.**

(5 marks)

17. If u $=\frac{1}{r}$ **and** $r^2=(x-a)^2+(y-b)^2+(z-c)^2$. **Prove that** $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0.$ **(4 marks)**

18. Verify Euler's theorem for the homogeneous function $u=x^3+y^3+z^3+3xyz.$ **(4 marks)**

19. If $u=\log(\tan x+\tan y+\tan z)$ **then show that** $\sin 2x\frac{\partial u}{\partial x}+\sin 2y\frac{\partial u}{\partial y}+\sin 2z\frac{\partial u}{\partial z}=2.$ **(4 marks)**

20. Find all the maxima, minima and saddle points of the function F $(x,y)=x^2+3xy+3y^2-6x+3y-6.$

(5 marks)

21. Find the volume of the prism whose base is the triangle in the xy-plane **bounded by the x-axis and the lines y = x and x = 1 and whose top lies in the plane** $f(x,y)=3-x-y.$

(4 marks)

22. Evaluate $\int_0^1\int_0^{2-x}\int_0^{2-x-y} dz\,dy\,dx.$ **(4 marks)**

23. Find a quadaratic **approximation of** $f(x,y)=e^x\cos y$ **by Taylors formula at the origin. (5 marks)**