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SECOND YEAR B.Sc. DEGREE EXAMINATION
SEPTEMBER/OCTOBER 2009

Part III Statistics (Subsidiary)

Paper II PROBABILITY DISTRIBUTION AND STATISTICAL INFERENCE

(2000 Admissions)

Time : **Three Hours** **Maximum : 60 Marks**

1. **The first four moments of a r.v. X about X = 4 are 1, 4, 10 and 45. Show that the mean is 5 and variance is 3 and r_3 and μ_4 are 0 and 26 respectively.**

(5 marks)

2. **Define m.g.f. of a r.v. Mention any two limitations of m.g.f.** **(5 marks)**

3. X **and Y have bivariate distribution given by :**

$P(X = x \cap Y = y) = \frac{x + 3y}{24}$ **where (x, y) = {(1, 1), (1, 2), (2, 1), (2,2). Find the conditional mean of X given y = 2.**

(5 marks)

4. **Let X have Poisson distribution with parameter** > 0. **Prove that :**

$1 \quad {}_{+1} \qquad \underline{d} \quad {}_{r}$

(5 marks)

5. **Obtain the m.g.f. of geometric distribution and hence obtain the mean and variance.**

(5 marks)

6. **Let X be a continuous r.v.** having p.d.f,

$$f(x) = \begin{cases} x; & 0 \le x \le 1 \\ 2 - x & 1 < x < 2 \\ 0; & \text{elsewhere} \end{cases}$$

Find $M_1(t)$.

(5 marks)

7. Let X_i $(i = 1,2, \ldots, n)$ **be** n **independent normal** variates **with mean** μ_i **and variance** **respectively.**

Then show that $\sum_{i=1} a_i X_i \sim N \left(\sum_{i=1} a1 \qquad \sum_{i=1} a_i^2 \sigma_i^2 \right)$

Hence deduce that $\bar{X}$ **N** $\sigma 2$ **n y**

(5 marks)

Turn over

8. State Chebychev's inequality. If a r.v. X has mean 5 and variance 3, what is the least value o

P $- 5| < 3\}$?

(5 marks

9. State and prove Bernoulli's Law of Large Numbers. (5 marks)

10. State Lindberg-Levy form of CLT. Decide whether CLT holds for the sequence of independent random variables with distribution defined as :

$$P(X_r = 1) = P_r \text{ and } P(X, = 0) = 1 - P_r.$$

(5 marks)

11. Define t - variate. Derive its p.d.f. (5 marks)

12. Let X_1, X_2, X_3, X_4, be independent random variables such that :

$$E(X_i) = \mu \text{ and } V_{u,}(X_i) = a^{-2}.$$

If $= \frac{+X2 + X3 + X4}{4}$ and

$$Z = X1 \frac{+2X_2 + X3 - X4}{4}$$

examine whether Y and Z are unbiased for What is the efficiency of Y relative to Z.

(5 marks)

13. State factorisation theorem on sufficiency. If x_1, x_2, x_n is a random sample from a distribution :

$$f(x, 0) = (1 - 0)^{1,}, \; x = 0, 1$$
$$= 0, \text{ elsewhere},$$

show that $y = x_i + x_2 + \quad + x_n$ is a sufficient statistic for 0.

(5 marks)

14. The sample values from population with p.d.f. : $f(x) = (1 + 0)\, x^0$, $0 < x < 1$ are given below :

0.4, 0.3, 0.6, 0.8, 0.4 .

Find the estimator of 0 by the method of moments.

(5 marks)

15. Define "Confidence interval". Obtain the 95 % confidence interval for "μ" of the Normal population N $\sigma^{\hat{}}$), where σ is known.

(5 marks)

16. Define critical regions and state Neyman-pearson Lemma. (5 marks)

17. Discuss the large sample test for population mean (S. D. is known). (5 marks)

18. Explain how you will test the equality of variances of two normal populations. (5 marks)