

SECOND YEAR B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2009

Statistics (Subsidiary)

Paper II PROBABILITY DISTRIBUTION AND STATISTICAL INFERENCE

(2000 admission onwards)

Time : Three Hours —————

Maximum : 60 Marks

Each question carries 5 marks.

1. If $\mu_1^1 = 1, \mu_2 = 2, \mu_3 = 3$ and $\mu_4 = 4$, find μ_2, μ_3 and μ_4 .
2. Define moment generating function of a r.v. X. Show that

$$\mu_r' = \left. \frac{d}{dt^r} \phi(t) \right|_{t=0}$$

3. Let X and Y have joint p.d.f.

$$\begin{aligned} & 2, \quad 0 < x < y < 1 \\ & 0, \quad \text{otherwise} \end{aligned}$$

Show that the conditional mean and variance of X gives Y = y are $\frac{y}{2}$ and $\frac{y^2}{12}$ respectively.

4. If the recurrence relation for the central moments of Poisson distribution is $\mu_{r+1}' = \frac{r}{r-1} \mu_r'$, obtain hence obtain β_1 and β_2 and interpret β_1 and β_2 .
5. State and prove the "Lack of Memory Property" of Geometric distribution.
6. Define Triangular distribution and draw the graph of the p.d.f.
7. Find the m.g.f. of Gamma distribution and examine whether Gamma distribution possesses additive property.
8. State Chebychev's inequality. Show that for a geometric distribution $P(x) = \dots = 1, 2, 3, \dots$

$$P\{|X - 2| \leq 2\} > \frac{1}{2}.$$

9. Let X_i assume values i and $-i$ with equal probabilities. Show that the law of large numbers cannot be applied to the independent variables X_1, X_2, \dots

Turn over

- u. tate Lindberg – Levy form or LPL 1. Let A_1, A_2, \dots Poisson random variables with parameter $\lambda_i = 1$. Use CLT to estimate

$$P(10 \leq S_{100} \leq 50),$$

$$\text{where } S_{100} = X_1 + X_2 + \dots + X_{100}.$$

1. Define F-variate. Derive its p.d.f.
12. Define “unbiasedness”. If X_1, \dots, X_n is a random sample from a normal population $N(\mu, \sigma^2)$, Show that $t = \frac{1}{n} \sum x_i^2$ is an unbiased estimator of $\mu^2 + \sigma^2$.
13. Let x_1, x_2, x_3, x_4 be a random sample from $N(\mu, \sigma^2)$ population. Find the efficiency of $T = \frac{1}{7} [x_1 + 3x_2 + 2x_3 + x_4]$ relative to $\bar{X} = \frac{1}{4} [x_1 + x_2 + x_3 + x_4]$ Which estimator is relatively more efficient? Why?
14. If X_1, X_2, \dots, X_n is a random sample obtained from the density function :

$$f(x, \theta) = \begin{cases} 1, & 0 \leq x \leq \theta \\ 0, & \text{elsewhere} \end{cases}$$

Show that the sample mean \bar{x} is consistent for θ

15. What are confidence intervals? Obtain 95 % confidence interval for population proportion based on a large sample.
16. Define (i) Type I error ; (ii) Type II error ; (iii) Level of significance ; (iv) Critical region.
17. Discuss the Chi-square test from goodness of fit.
18. State any two assumptions involved in t-test. What are the various steps involved in testing $H_0 : \mu = \mu_0$, relating to normal population, $N(\mu, \sigma^2)$, when σ is unknown and sample size is small.