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SECOND YEAR B.Sc. DEGREE EXAMINATION, MAY 2010

Part III—Statistics— Subsidiary

Paper II—PROBABILITY DISTRIBUTIONS AND STATISTICAL INFERENCE

(2000 Admissions onwards)

Time :Three Hours | Maximum : 60 Marks

1. Define mathematical expectation of a random variable. If X and Y are two discrete random variables and a and b are constants ,show that E $(aX + bY) = a\,E(X) + b\,E(Y)$.

(5 marks)

2. If $\mu_1^{'} = 1$, μ4 $= 2$, $\mu_3 = 2$ and $\mu_4^{'} = 3$, obtain the values of , μ_2, μ_3 and μ_4. What is the nature of skewness of the distribution ?

(5 marks)

3. If X and Y have joint p.d.f.

$$f(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{otherwise,} \end{cases}$$

show that the conditional mean and variance of X given Y = y are $\frac{y}{2}$ and $\frac{y^2}{12}$ respectively.

(5 marks)

4. If X has a discrete uniform distribution over the integers 0, 1, 2, 3 and 4, find E (X) V (X).

(5 marks)

5. Derive the m.g.f. of geometric distribution and hence obtain the mean of the distribution.

(5 marks)

6. If $X_1, X_2, \ldots, X_n$ are independent random variables, X_i having an exponential distribution with parameter θ_i, $i = 1, \ldots, n$, then show that $Z = \text{Min}(X_1, X_2, \ldots X_n)$ has exponential distribution with parameter $\underset{i=1}{} \theta_i$.

(5 marks)

7. State and prove additive property of normal variables. (5 marks)

8. State Chebychev's inequality. If denotes the sample mean of a random sample of size 'n' drawn from a population having mean ' and variance = 1, find the minimum value of 'n' such that P {IX. − μ I <1} > 0.95.

(5 marks)

Turn over

9. Define the concept of "Convergence in probability". Also state and prove Bernoulli's Law of large numbers.

(5 marl

10. State and prove De-Moivre Laplace form of Central Limit Theorem. (5 marl

11. Define a Chi-square variate with n degrees of freedom and obtain the mean and variance. Al mention any one use of Chi-square distribution.

(5 marks)

12. Define 'unbiasedness'. If $x_1, x_2, \ldots x_l$ is a random sample from. a normal population N ,1),

show that $t - \frac{x_i^2}{i=1}$ is an unbiased estimator of μ +1. (5 marks)

13. Let x_1, x_2, x_3, x_4 be a random sample from $N(\mu, \sigma)$ population. Find the efficiency of

$= \frac{1}{7}(\ + 3x_2 + 2x_3 + x_4)$ relative to $X = \frac{1}{} \sum_{i=1}$. Which is relatively more efficient ? Why ?

(5 marks)

14. Let X be a gamma variate with p.d.f.

$\frac{\beta^{\alpha}}{a}$

Obtain the moment estimators of a and

(5 marks)

15. Let $x_1, x_2, \ldots, x_n$ be a random sample from N σ)

(i) Obtain 100 (1 – a)% confidence interval forμ when σ is known.

(ii) Obtain 100 (1 – a)% confidence interval for υ .

(5 marks)

16. Define Level of significance and 'power' of a test. Given the frequency function

$$f(x, \theta \quad \begin{cases} \frac{1}{0} & 0 < x < 0 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate the level of significance and power associated with testing $H_o : O = 1.5$ against $H_1 : O = 2.5$ by means of a single observed value x, if the critical region is chosen as $x > 0.8$

(5 marks)

17. Explain the Chi-square test for independence of two attributes. (5 marks)

18. Explain the test procedure associated with testing $H_0 : p = 0$, where p refers to the correlation coefficient of a bivariate normal distribution.

(5 marks)