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SECOND YEAR B.Sc. DEGREE EXAMINATION, MAY 2010

Part III Subsidiary

Mathematics Paper I—ANALYTICAL GEOMETRY AND CALCULUS

(2001 Admissions)

Time : Three Hours | Maximum : 65 Marks

Maximum marks that can be scored from Unit I is 20, *Unit II is* 30 *and Unit III is* 15.

Unit I (Analytical Geometry)

(Maximum Marks : 20)

Transform the equation $x^2 - z^2 = 4$ to cylindrical and spherical polar coordinates. (4 marks)

Prove that, in general, three normals can be drawn to a parabola from any point in its plane and that sum of the ordinates of the feet of the normals is zero.

(5 marks)

3. Find the centre, foci, directrices and latus rectum of the ellipse $9x^2 - 25y^2 + 18x + 100y - 116 = 0$.

(8 marks)

4. Find the equation of the hyperbola whose asymptotes are $2x - y - 3 = 0$ and $3x + y - 7 = 0$.

(4 marks)

5. Show that the equation $l = r(1 + e\cos\theta)$ represents a conic with semilatus rectum l and eccentricity e.

(5 marks)

6. Find the equation of the circular cylinder haves for its base the circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$.

(4 marks)

Unit II (Differential Calculus)

(Maximum Marks : 30)

7. **If** $y = (\sin^{-})^2$, prove that $(1-X^2)\, y_{n+2} - (2n+1)xy_{n+1} - n^{\hat{}}\, y_n = 0$. (5 marks)

8. If R..) $= 2x - x^{\hat{}}$, $a = 0, b = 1$, determine c such that $f(b) = f(a) + (b-a)f'(c)$, $a < c < b$.. (4 marks)

9. Evaluate $\lim_{x \to 0} (\cos x)^{1/x^2}$. (4 marks)

10. Using Maclaurin's series, prove that log sec x $= \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \ldots$ (5 marks)

Turn over

11. **Find the evolute $y^2 = 4x$.** **(5 marks)**

12. **If** $u = \log\left|\frac{x^3 + y^3}{X + y}\right.$ **, prove that** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$. **(4 marks)**

13. **If it** $= x^y$**, find** $\frac{\partial^2 u}{\partial x \partial y}$ **(4 marks)**

14. **If** $= r\sin\theta\cos\varphi$, $y = r\sin\theta\sin 4)$, $z = r\cos 0$, **prove that** $\frac{\partial(x,y,z)}{} = r \sin\theta$. **(4 marks)**

15. **If** $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ **prove that** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(4 marks)**

16. **Find the asymptotes of $x^3 - 4y^3 + 3x^2 + y - x + 3 = 0$.** **(6 marks)**

Unit III (Integral Calculus)

(Maximum Marks : 15)

17. **Obtain the approximate value of** $\int_{x}^{3} dx$ **by Simpson's rule after dividing the range Into 4 equal parts.**

(5 marks)

18. **Find the length of the arc of the parabola** $y^2 - 4x$ **from the vertex to the point (4, 4).** **(5 marks)**

19. **Find the area of the surface of revolution of** $x = a\cos t$, $y = a\sin t$ **about the x-axis.**

(5 marks)

20. **Find the volume generated by revolving the region enclosed by The loop of** $2ay = x(x-a)^2$ **about the x-axis.**

(5 marks)

21. **Evaluate** $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dy\, dx$. **(5 marks)**