

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2014

(U.G.—CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Unit I

*Answer all twelve questions.
Each question carries $\frac{1}{4}$ weightage.*

1. **Tanh x** is equal to :

(a) $\frac{e^x - e^{-x}}{2}$ (b) $\frac{e^x + e^{-x}}{2}$

(c) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ (d) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$

2. Write **csch⁻¹ x** as a logarithmic function :

3. Find $\frac{d}{dx} \left(6 \sinh \frac{x}{3} \right)$.

4. Find $\int_0^1 \frac{1}{\sqrt{x}} dx$.

5. The nth term of the sequence

$$0, 3, 8, 15, 24, \dots$$

6. Find $\lim_{n \rightarrow \infty} \sqrt[n]{3n}$.

7. Find the sum of the series $\sum_{n=1}^{\infty} \frac{3^{n-1}}{6^{n-1}}$

8. Test the convergence of the series

$$1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \dots$$

Turn over

9. Define the absolute convergence of a series a_n .
10. Find the Cartesian equation of the curve $r = 6 \sin \theta$.
11. Write the polar equation of the hyperbola with $k = 2$ and $e = \frac{3}{2}$.
12. $f(x, y) = 100 - x^2 - y^2$. Find the level curve of $f(x, y) = 75$.

(12 x ¼ = 3 weightage)

Unit II

*Answer any nine questions.
Each question carries 1 weightage.*

13. Differentiate $\tanh^{-1} t^2$ with respect to t .
14. Find $\int \operatorname{sech}^{-1} \left(x - \frac{1}{2} \right) dx$.
15. Find $\int_0^2 \frac{dx}{1 - x^2}$.
16. Find the sum of the series $\sum_{n=1}^{\infty} \frac{6}{(2n-1)^2}$.
17. Does the series $1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \dots$ converge?
18. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^{n+1}}$ is absolutely convergent.
19. For what value of x does the power series $\sum_{n=1}^{\infty} x^n$ converge?
20. Find the Taylor series expansion of $f(x) = \ln(1+x)$ at $x = 0$.

21. Find the polar equation of the ellipse $4x^2 + 4y^2 = 36$.

22. Write the centre and radius of the circle $r + \cos \theta = 0$.

23. Find $\lim_{x \rightarrow \infty} (\ln(x^2 + y^2 + z^2))$.

24. Find $\frac{\partial f}{\partial x}$ at $(4, -5)$ if $f(x, y) = x^2 + 3xy + y$.

(9 x 1 = 9 weightage)

Unit III

*Answer any five questions.
Each question carries 2 weightage .*

25. Show that $\cosh^{-1} x = \ln \left(x + \sqrt{x^2 - 1} \right), x > 1$.

26. Find $\sqrt{0^2 + 50 + 6}$

27. Show that $\frac{1+21n}{9} + \frac{21+31n}{14} + \frac{31+41n}{2}$ diverges.

28. Find the points of intersection of the pair of curves $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$.

29. If $f(x, y) = x \cos y + y e^x$ find $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y^2}$

30. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s where $w = x^2 + y^2, x = r + s, y = r - s$.

31. Find the directional derivative of $f(x, y) = 3xy - y^2$ at $(5, 5)$ with direction of $A = 4i + 3j$.

32. Find the area that lies inside the circle $r = 1$ and outside $r = 1 - \cos \theta$.

(5 x 2 = 10 weightage)

Unit IV

*Answer any two questions.
Each question carries 4 weightage.*

33. (a) Evaluate $\int_1^{\infty} \frac{x+3}{x(x+1)} dx$

(b) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{4^n}{(2n)!}$.

34. Find the length of the cardioid $r = 1 - \cos \theta$.

35. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2).

(2 x 4 = 8 weightage)

