

**SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2015**

(CUCBCSS—UG)

Complementary Course

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

**Part A***Answer all questions,*

1. Define a smooth curve.
2. Write down the relation connecting  $\sin x$  and  $\sinh x$ .
3. Evaluate  $\int_0^1 \frac{dx}{\sqrt{x}}$ .
4. Give an example of a non-decreasing sequence.
5. State Sandwich theorem for the sequence.
6. Define absolute convergent sequence.
7. Find the equation for a hyperbola with eccentricity  $= 3/2$  and directrix  $x = 2$ .
8. What is the formula in polar co-ordinates for the area of the surface generated by revolving the curve about the x-axis.
9. Find the equation of the circular cylinder  $4x^2 + 4y^2 = 9$  in cylindrical co-ordinates.
10. Define level surface of  $f$ .
- ii Find  $\lim_{y \rightarrow 0} \frac{x - xy \pm 3}{x^2 + 5xy + y}$
12. Write down the chain rule for finding  $dw/dt$  if  $w = f(x, y, z)$  is differentiable and all  $x, y, z$  are differentiable functions of  $t$ .

(12 x 1 = 12 marks)

**Turn over**

**Part B**

Answer any **nine** questions.

13. Find the volume of the solid generated by revolving the region between the parabola  $x = y^2$  and the line  $x = 3$  about the line  $x = 3$ .
14. Find the length of the curve  $y = \frac{4\sqrt{2}}{3} x^{3/2}$
15. Find the area under the curve  $y = 1/\sqrt{x}$  from  $x = 0$  to  $x =$
16. Show that  $\lim_{n \rightarrow \infty} k = k$ .
17. Find  $\lim_{n \rightarrow \infty} 1/n$ .
18. Graph the set of points whose polar coordinates satisfy  $1 < r < 2, 0 < \theta < \pi/2$ .
19. Find all Cartesian equation of  $r \cos \theta = -4$ .
20. Find  $\lim_{(x,y) \rightarrow (1,1)} \frac{y - 2x + 2}{x - 1}$
21. Find  $f_x$  if  $f(x, y) = x^2 + 3xy + y$
22. Find the length of the curve  $r = 1 - \cos \theta$ .
23. Find the length of curve  $y = (x/2)^{2/3}$  from  $x = 0$  to  $x = 2$ .
24. Find the directrix of the parabola  $r = \frac{25}{10 + 10 \cos \theta}$

(9 x 2 = 18 marks)

**Part C**

Answer any **six** questions.

25. Compare  $\int_1^{\infty} \frac{dx}{x^2}$  and  $\int_1^{\infty} \frac{dx}{1+x^2}$

26. Find the lateral surface area of the cone generated by revolving the line segment  $x = 1 - y$ ,  $0 \leq y \leq 1$  about y-axis.
27. Find the length of  $y = x^{3/2}$  from  $x = 0$  to  $x = 4$ .
28. Find the radius of convergence of  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ .
29. Find the Taylor series generated by  $f(x) = x^3 - 2x + 4$  about  $a = 2$ .
30. Graph the curve  $r^2 = 4 \cos \theta$ .
31. Find the area of the region lie inside  $r = 1$  and outside  $r = 1 - \cos \theta$ .
32. Show that  $f(x, y) = \frac{2xy}{x^2 + y^2}$  has no limit as  $(x, y)$  approaches to  $(0, 0)$ .
33. Find  $dw/dt$  at  $t = 0$  if  $w = xy + z$ ,  $x = \cos t$ ,  $y = \sin t$ ,  $z = t$ .

(6 x 5 = 30 marks)

### Part D

Answer any two questions.

34. Write down the shell formula. Using this find the volume of the solid generated for the following problems.
- (a) The region bounded by  $y = \sqrt{x}$ , the x-axis and the line  $x = 4$  revolved about x-axis.
- (b) The region in the first quadrant bounded by  $y = x^2$ , y-axis and the line  $y = 1$  revolved about  $x = 2$ .
35. Define radius and interval of convergence. Investigate the convergence of  $\sum_{n=0}^{\infty} \frac{2^n}{n!} x^n$ ,  $n \geq 1$ .
36. (a) Write the chain rule and draw the tree diagram for finding  $\frac{\partial w}{\partial r}$  as  $\frac{\partial w}{\partial s}$  if  $w = x^2 + y^2$   
 $x = r - s$ ,  $y = r + s$ .
- (b) Using Implicit differentiation, find  $dy/dx$  if  $x^2 + \sin y - 2y = 0$ .

(2 x 10 = 20 marks)