Reg. No. . . . .

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL/MAY 2013

(CCSS)

MM 2C 02—MATHEMATICS

Time : Three Hours | **Maximum : 30 Weightage**

I. Objective type questions. Answer *all* questions, weightage 'A each :

1 Show that $\cosh 2x = \cosh x \quad \sinh^{-} x$.

2 Investigate the convergence of $\int \frac{1}{x} dx$

3 Find $\lim_{n-3.} \frac{\ln n^2}{n}$

4 Define the convergence of a sequence.

5 Find a formula for the nth term of the sequence 1, – 4, 9, – 16, 25, . .

6 Define the alternating series test.

7 The least upper bound of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots \frac{}{n+1}, \ldots$ is ————

8 Graph the set of points whose polar co-ordinates satisfy the conditions $r \quad 0$ and $\theta = \frac{\pi}{4}$

9 Show that the point $\left(, \frac{\pi}{2} \right)$ lies on the curve $r = 2 \cos 20$.

10 Find $\frac{at}{}$ at (4, -5) if $f(x, y) = x^2 + 3xy \; y - 1$.

11 Find $\frac{\partial f}{}$ if $f(x, y) = y \sin . xy$.

12 If x, y and z are independent variables and $f(x, y, z) = x \sin (y + 3z)$ find $\frac{}{\partial z}$.

(12 x ¼ = 3 weightage)

II. Short answer type questions. Answer all *nine* questions, weightage 1 each :

13 Find the derivative of $y \quad \tanh$ with respect to t.

Turn over

14 Show that the series $\sum_{n=1} 2$ diverges.

15 Test the convergence of $\sum_{n=1} \frac{n+1}{n}$.

16 Examine the convergence of $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$.

17 For what values of x does the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1}$ converges.

18 Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.

19 Define the gradient of $f(x, y)$.

20 Find an equation for the tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at (-2, 1).

21 Find $\frac{\partial z}{\partial x}$ if the equation $yz \ \ \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivatives exists.

(9 x 1 = 9 weightage)

III. Short essay questions. Answer any *five* questions, weightage 2 each

22 Compare $\int_1 \frac{dx}{x^2}$ and $\int \frac{dx}{1+x^2}$ with the limit comparison test.

23 Does $\sum_{n=1} \frac{\ln n}{n^{3/2}}$ converge ?

24 Prove that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

25 Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

26 Verify that $W_{xy} = W_{yx}$ if $W = e^x + x \ln y + y \ln x$

27 Find $\frac{\partial W}{\partial r}$ when r = 1, s =1 if $W = (x + y + z)^2$, $x = r - s$, $y = \cos(r + s)$, $z = \sin(r + s)$.

28 Find the length of the asteroid $x = \cos^3 t, y = \sin^3 t, 0 < t < 2\pi$

(5 x 2 = 10 weightage)

29 Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$ (p is a real number) converges

if p >1 and diverges if p

30 Find the Taylor series and the Taylor polynomial generated by $f(x) = ex$ at x = 0.

31 Find the linearization, $L(x, y)$ of $f(x, y) = e$ cos y P_o (0, 0) and find an upper bound

for I E | of the error in the approximation $f(x, y)$ $, y)$ over the rectangle

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(2 x 4 = 8 weightage)