C 25709 (Pages 4) Name .....

Reg. No..................

# SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics (Elective Course)

## MM 6B 13 (E02)—LINEAR PROGRAMMING AND GAME THEORY

Time : Three Hours Maximum : 30 Weightage

### Part I

*Answer* **all** *questions.*

1. Maximize $Z = x_1 + x_2^2$, subject to $x_1 - x_3 = 3$ and $x_2$ 2 is a ———
   - (a) Linear Programming problem.
   - (b) Quadratic programming problem.
   - (c) Transportation problem.
   - (d) Assignment problem.
2. Define a convex set.
3. What is surplus variable ?
4. Which of the following is not a convex set in $R^2$ ?
   - (a) $\{(x, y) / x + 2y = 3\}$.
   - (b) $\{(x, y) / x^2 + y^2 \le 1\}$.
   - (c) $\{(x, y) / a < x < b\}$.
   - (d) $\{(x, y) / x^2 + y^2 = 1\}$.
5. Are the vectors a = (1, 2, 3), b = (-6, 0, 2) are Orthogonal ?
6. Which of the following sets form a basis of $R^2$ ?
   - (a) $\{(2, 0)\ (3, 0)\}$.
   - (b) $\{(0, -1)\ (0, 1)\}$.
   - (c) $\{(2, 0)\ (0, 2)\}$.
   - (d) $\{(0, 0)\ (0, -2)\}$.
7. Define support of a set in $E^n$
8. Find the convex null of the $, y \in R\ / x\ + 1\ = 3\}$.
9. Find a basic solution of the following system with $x_3$ as a non basic variable $2x_1\ x_2 + 3x_3 = 3$ ;

   $+2x_2 - x_3 = 4$.
10. Define a saddle point of a two person zero sum game.

Turn over

11. Find the dual of Maximize $Z = 3x_1 + x_2$, $2x_1 + 3x_2$ 5; $x_1 + x_2 \geq 3$, $x_1$ 0, $x_2 \geq 0$

12. Express (−1, 2) as a linear combination of (2, 0) and (0, 2).

(12 x ¼ = 3 **weightage**)

**Part II**

*Answer* **all** *questions.*

13. Find the convex null of the set $(1, 2), (2, 3)\}$.

14. Show that $\bar{a} = (1, 2, 1)$; $b = (2, 3, 0)$; c = (1, 2, 2 are linearly independent in $E^3$.

15. Prove that every **hyperplane** in $R^n$ is convex.

16. Find a basic solution of the system

$$x_1 + 2x_2 - x_3 + x_4 = 4$$

$$x_1 - x_2 + 2x_3 - x_4 = -2$$

17. Transform the following into Standard form :

Maximize $Z = 2x_1 + 3x_3$

subject to $x_1 + x_2 < 1$

$3x_1 + x_2$ 4

x 0; $x_2$

18. Convert the following into a maximization problem

Minimize $Z = 4x_1 + 3x_2$

subject to $x_1 + 2x_2$ 8

$3x_1 + 2x_2$

$\geq 0$; $x_2 \geq$

19. Obtain the dual of ~~Maximize~~ $Z = x_1 - x_2 + 3x_3$, subject to $x_1 + x_2$ $x_3 \leq 10$; $2x_1 - x_3$ 2, $2x_1 - 2x_2 + 3x_3$ 6.

20. Define a loop in a transportation problem.

21. Define **maximin** principle in a two person zero sum game.

x 1 = 9 **weightage**)

## Part III

*Answer* **any five** *questions.*

**22. Draw the feasible space of the following in equations:**

$x_1 \quad {}^2x_2 \quad 7,\ x_1 \quad x_2 \quad 4:\ x_1 > 0;\ x_2 > 0.$

**23. Show that** $X = \{(x_1, x_2) / x_1 - 2x, = 2\}$ **is a convex set in** E .

**24. Show that set of all feasible solutions of a system of equations** $AX = b$ **is closed convex set.**

**25. Given the system:**

$2x_1 \ x_2 + 2x_3 = 10,\ x_1 + 4x_2 = 18$ **and** $x_1, x_2 \geq O$. **Obtain a basic feasible solution starting from (2, 4, 5).**

**26. Using north-west corner rule find an initial basic feasible solution of the transportation problem.**

| | $D_1$ | $D_2$ | $D_3$ | |
|---|---|---|---|---|
| $Q_1$ | 3 | 8 | 7 | 10 |
| $Q_2$ | 6 | 5 | 8 | 5 |
| | 6 | 5 | 4 | |

**27. Solve the following 2 x 2 game**

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 4 & 2 \\ 1 & \end{bmatrix} \end{array}$$

**28. Show that** $(1, 2, -1)$, $(0, 1, 1)$ **and (1,1,1) generate the vector space** $R^3$.

**(5 x 2 = 10 weightage)**

## Part IV

*Answer any two questions.*

**29 Use Simplex method to solve:**

Minimize $Z = \quad 3x_2 + 2x_3$

subject to $\quad 3x_1 - x_2 + 2;$

$2x_1 + 4x_2 \quad 12$

$4x_1 + 3x_2 + 8x_3 \leq 10$

$, x_2, x_3 \ O.$

**Turn over**

30. Solve the transportation problem :

| | $D_1$ | $D_2$ | $D_3$ | Availability |
|---|---|---|---|---|
| $S_i$ | 5 | 1 | 8 | 12 |
| $S_2$ | 2 | 4 | 0 | 14 |
| $S_3$ | 3 | 6 | 7 | 4 |
| Requirement | 9 | 10 | 11 | |

31. **Solve the following game :**

$$\begin{bmatrix} 1 & -3 & 2 \\ -4 & 4 & -2 \end{bmatrix}.$$

**(4 x 2 = 8 weightage)**