Reg. No. ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2014

(U.G.—CCSS)

Mathematics

MM 6B 10—COMPLEX ANALYSIS

Tin : Three Hours Maximum : 30 Weightage

Section A

Answer all **twelve** *questions.*

What is the imaginary part of $z^2 + 5$?

What is the value of $\lim_{z \to \infty} \left(\frac{5z + i}{z + 4} \right)$,

. The real and imaginary parts of an analytic functions are ______ functions.

$|e^z| =$ ______

(a) e. (b) e^x.

(c) e^x. (d) e^2.

– ______

(a) -1. (b) 1.

(c) 0. (d) e.

6. Express sin x in terms of e^{ix}

7. What is the parametric form of the circh $|z| = 2$?

8. Every bounded entire function is ______

9. The region of convergence of $1 - z + z^2 - z^3 + \ldots$ is ______

10. What is the value of $\int_{|z|=2} (z^2 + 5)\, dz$

Turn over

11. For the function $f(z) = \frac{\sin z}{\ }$, $z = 0$ *is*

(a) Removable singular point. (b) Pole of order 1.

(c) Pole of order 2. (d) Essential singular point.

12. Identify the poles of $\frac{5z}{z^2}$

(12 x ¼ = 3 weightage)

Section B

Answer all **nine** *questions.*

13. Show that $f\,(z)$ does not exist at any point for $f(z) = z$.

14. If $f(z)$ and $\overline{f(z)}$ are both analytic in a domain D, prove that $f(z)$ is a constant throughout D.

15. Find all values of z such that $e^z = -2$.

16. Find the principal value of (– .

17. State Cauchy–Goursat theorem.

18. Deduce Cauchy's inequality from the extension of Cauchy's integral formula.

19. State Laurent's theorem.

20. Discuss the nature of singularity of $e^{1/z}$ at $z = 0$.

21. For the function $f(z) = \frac{\sin z}{z^4}$ determine the order of the pole at $z = 0$ and the corresponding residue.

(9 x 1 = 9 weightage)

Section C

Answer any **five** *questions.*

22. Show that $u(x, y) = y^3 - 3x^2 y$ is a harmonic function and find a harmonic conjugate $v(x, y)$ of u.

23. If $f(z) = u(x, y) + i\,v(x, y)$ is analytic in a domain D, prove that the families of level curves $u(x, y) = c_1$, and $v(x, y) = c_2$ are orthogonal.

24. Find all roots of the equation $\cosh z = \frac{1}{2}$.

25. Evaluate $\int_C \frac{dz}{(z^2+4)^2}$ where C is the circle $|z-i| = 2$.

26. State and prove fundamental theorem of algebra.

27. Expand e^z as a Taylor series about $z = -1$ and state the region of validity of the expansion.

28. State and prove Cauchy's residue theorem.

(5 x 2 = 10 weightage)

Section D

Answer any **two** *questions.*

29. State and prove Cauchy's integral formula.

30. Expand $\frac{z}{(z-1)(z-3)}$ as

(i) Maclaurin's series and

(ii) Laurent series in $0 < |z-1| < 2$.

31. Using Cauchy's residue theorem evaluate :

$$\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}, \quad -1 < a < 1.$$

(2 x 4 = 8 weightage)