Reg. No....................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2014

(U.G.—CCSS)

Mathematics

MM 6B 12—NUMBER THEORY AND LINEAR. ALGEBRA

Time Three Hours Maximum : 30 Weightage

I. Answer all *twelve* questions

1 $a(5) =$

2 The value of $\sum_{n=1}^{6} \sigma(n) =$ __________

3 If p is a prime and $k > 0$ then $\phi\left(p^{\cdot}\right. \;=$ ______________

4 If a, b, c are integers and $gcd\ (a, b, c) = 1$ then $gcd\ (a, c) =$ ____________

5 If A is a non-singular matrix of order n then the rank of A is ____________

6 The system AX = 0 in n unknows has a trivial solution if :

(a) $\rho(A) > n$. (b) $p(A) = n$.

(c) $\rho(A) < n$. (d) None of these.

7 State Cayley-Hamilton theorem.

8 Define Nullity of a matrix.

9 If A is a matrix of order m x n and R is a non-singular matrix of order m then

____

10 If N is a positive integer then $\sum_{n=1}^{N} \sigma(n)$ $\quad \overset{N}{\underset{n=1}{}}$ ____________

11 State Chinese Remainder theorem.

12 If $ca \quad cb$ (mod n) and $gcd\ (c, n) = 1$ then a __________ (mod n).

(12 x $\frac{1}{4}$ = 3 weightage)

Turn over

II. Answer all *nine* questions :

13 When two integers a and b are said to be relatively prime ?

14 Find lcm (306, 657).

15 State Fermat's theorem.

16 Prove that the function μ is a multiplicative function.

17 Show that no skew-symmetric matrix can be of rank 1.

18 If $A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ find p (A).

19 Define Null space of a matrix.

20 Find the remainder when $1! + 2! + 3! + \quad + 99! + 100!$ is divided by 12.

21 Prove that if $gcd\ (a, b) = d$ then $gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

(9 x 1 = 9 weightage)

III. Answer any *five* questions from seven

22 Prove that the fourth power of any integer is of the form 5k or 5k + 1.

23 Let a and b be integers, not both zero prove that a and b are relatively prime if and only if there exist integers x and y such that $1 = ax + by$.

24 Prove that for positive integers a and b $gcd\ (a, b)\ lcm\ (a, b) = ab$.

25 Show that 8 divides $7^{2n+1} + 1$.

26 Using Wilson's theorem prove that $18! + 1\ 0$ (mod 23).

27 Find non-singular matrices P and Q such that PAQ is in the normal form given

$$A = \begin{pmatrix} & 1 & 1 \\ 1 & 2 & 3 \\ & -1 & 1 \end{pmatrix}$$

28 Show that the characteristic roots of an idempotent matrix are either zero or unity.

(5 x 2 = 10 weightage)

Iv. Answer *two* questions from three

29 State and prove Chinese Remainder theorem.

30 State and prove Fundamental theorem of Arithmetic.

31 Using Cayley-Hamilton theorem show that $A^3 - 6A^2 + 11A - 61 = 0$ where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$

and hence find A^{-1}.

(2 x 4 = 8 weightage)