80027 (Pages : 3) Name..............................

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SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.–CCSS)

Core Course—Mathematics

MM GB 11—NUMERICAL METHODS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions from this part.*

1. If $f(x)$ is continuous in [a, b] and $f(a)$ and $f(b)$ are of opposite signs then which of the following is true:
   (a) There exists exactly one root of $f(x) = 0$ between a and b.
   (b) There exist at least one root of $f(x) = 0$ between a and b.
   (c) There exist at most one root between a and b.
   (d) There is no root between a and b.

2. Find the second approximation of a real root of $x^2 - 2x - 5 = 0$ using bisection method.

3. Write the Newton-Raphson formula.

4. Define the central difference operators.

5. Write the Newton's backward difference formula.

6. Write the Lagrange polynomial of degree 2.

7. Write the general form of the unit lower triangular matrix.

8. Find the characteristic equation of the matrix

$$\begin{vmatrix} 1 & 6 & f \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$$

9. $y' = x + y^2$ with y (0) = 1. Find the second approximation $y^{(2)}$ using Picard's method.

10. Write Simpson's $\frac{1}{3}$ rule.

11. In Adams-Moulton method ________ formula is used to derive Predictor-corrector formula.

12. Write the Milne's corrector formula.

(12 x $\frac{1}{4}$ = 3 weightage)

Turn over

Part B

Answer **all** *questions from this part.*

13. Find the second approximation of a real root of the equation $x^3 - 4x - 9 = 0$ using bisection method.

14. Find an iteration formula used to find a root of the equation $x \sin x + \cos x = 0$ using Newton-Raphson formula.

15. Using Ramanujan's method obtain the first two convergents of the equation $x + x^3 = 1$.

16. Prove that $E = e^{hD}$ where D is the differential operator.

17. Write Bessel's interpolation formula.

18. Fine the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$.

19. Evaluate $\int_0^1 \frac{1}{1+x}\,dx$ correct to three decimal places Simpson's $\frac{1}{3}$ rule taking $h = 0.5$.

20. Find the unit lower triangular matrix L in the LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}$$

21. $\frac{dy}{dx} = 1 + xy$ and $y(0) = 1$, obtain the Taylor series for y (x).

(9 x 1 = 9 weightage)

Part C

Answer any **five** *questions from this part.*

22. Find a real root of the equation $x^3 - x^2 - 2 = 0$ by Regula-Falsi method.

23. Using method of separation of symbols, show that

$$\Delta^n u_{x-n} = u_x - n\, u_{x-1} + \frac{n(n-1)}{2} u_{x-2} \ \ldots \ (-1)^n u_{x-n}.$$

24. The Population of a town in decennial census was given below. Estimate the population for the year 1925 :

Year *(x)*	1891	1901	1911	1921	1931
Population (y) (in thousands) :	46	66	81	93	101

25. Using Lagrange interpolation formula, express the function $\frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$ as sums of partial fractions.

26. By Gauss elimination method solve the system of equations

$5x\ y - 2z = 142,\ x - 3y\ z = -30,\ 2x - y - 3z = -50.$

27. Determine the largest eigen value and corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

28. Apply Runge-Kutta method to find an approximate value of y for x = 0.1 taking $h = 0.1$, if

$\frac{dy}{dx} = x + y$, y(0)=1.

(5 x 2 = 10 weightage)

Part D

Answer any two questions from this part.

29. From the following table, find the value of e^{117} using Gauss's forward formula.

x	: 1.00	1.05	1.10	1·15	F20	F25	1.30
ex	: 2·7813	2.8577	3.0042	3·1582	3.3201	3.4903	3.6693

30. Solve the system of equations using factorization method :

$x + 2y + 3z = 14,\ 2x + 5y + 2z = 18,\ 3x + y + 5z = 20.$

31. Solve the Initial value problem $\frac{dy}{dx} = 1 + xy$, y (0) =1 for x = 0·4 by using Milne's method. Given that

x	0.1	0.2	0.3
y	1·105	1·223	1.354.

(2 x 4 = 8 weightage)