Reg. No...................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 09—REAL ANALYSIS

Time : Three Hours | Maximum : 30 Weightage

Part A

Answer all *twelve* questions :

1 State True *or* False "$f(x) = \frac{1}{x}$ has neither an absolute maximum nor an absolute -minimum on the set (0, 00) ".

2 ~~Does 4 sin x~~ = x has a positive solution in $\left(\frac{\pi}{2}, \pi\right.$.

3 Give an example to a uniformly continuous function on [0, b] ; b

4 Find the norm of the partition p = (0, 2, 3, 4).

5 Give an example to function which is Riemann integrable in [a, b].

6 State True *or* False "Every continuous function is integrable".

7 Define point-wise convergence of a sequence of function $\{f_n($

8 Show that $fn(x)$ $\frac{1}{x+n}$ in uniformly convergent in [0, b] ; b >

9 Define uniform norm of a bounded function on A c R and $\phi : A \to R$

10 Define an improper integral.

11 Define Beta function.

12 If n is a positive integer value of $n + 1$ *is* ____________

(12 x $\frac{1}{4}$ = 3 weightage)

Turn over

Part B

Answer all **nine** *questions.*

13 Define absolute maximum and absolute minimum of f A R ; A $\subseteq$ R.

14 State maximum—minimum theorem.

15 Show that the function $f(x) = \begin{cases} 0, \text{ when x is rational} \\ 1, \text{ when x is irrational} \end{cases}$ is not integrable on any interva

16 If f (x) < g (x) V x $\in$ [a, b], then show that g dx a a .

17 Use fundamental theorem of calculus to evaluate $\int_a x$ x.

18 Show that G = x" (1— x) for x t A = [0, 1] converges uniformly to g (x) = 0.

19 Show that $1 + x + \frac{x2}{2!} + \frac{x3}{3!} + \ldots$ is uniformly convergent in [– 1 1].

20 Examine the convergence o ∞ dx

21 Express $\int^{1} \frac{x^{2}}{-x^{5}} dx$ as a Beta function.

(9 x 1 = 9 weightage)

Part C

Answer any **five** *questions from* 7.

22 State and prove Intermediate value theorem.

23 Define a Lipschitz function. Also prove that a Lipschitz function $f : A \rightarrow R$ is uniformly continuous on A.

24 Show that $f(n) = x^2$ Riemann integrable on [0, k].

• 25 If $f : [a, b] \rightarrow$ R is monotonic on [al)] then prove that $f E$ R $[a, b$.

26 State and prove Weierstrass M-test.

27 If $\int_a (x)| \, dx$ converges, show that $\int_a^\infty f(x) \, dx$ converges.

28 Using Beta functions prove that :

$$\frac{x}{- n^4} dx \qquad \int_0 \frac{dx}{\sqrt{1 + n^4}} = 4\sqrt{2} \cdot$$

(5 x 2 = 10 weightage)

Part D

Answer any two from four questions.

29 (a) Let I = [a, b] be a closed bounded interval and let $f : I \rightarrow$ R be continuous on I. Prove that f *is* bounded on I.

(b) Define a step function and give an example.

30 Prove that if I is Riemann integrable on [a, b1 then it is bounded on [a, b].

31 (a) Test for uniform convergence $\sum^{\infty} \frac{\sin nx}{n p}$ for $p > 1$.

(b) Prove that $\int_0 e^{-ax}$ x dx $\frac{n}{a}$ where a > 0, n > 0.

(2 x 4 = 8 weightage)