U 40395 (Pages : 3) Name....................................

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 10—COMPLEX ANALYSIS

ne : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *the questions.*

1. What is the value of :

$$\lim \frac{2z+i}{z+1}$$

Find the imaginary part of $z + i$.

$(z) = u(x,y) + iV(x,y)$ is analytic in a domain D if and only, V is ______________ of u.

What is the real part of e^z ?

What is the period of sin z ?

- Express cos x in terms of e^{ix}.

7. The value of $e^{i\pi}$ is

(a) 1. (b) e

(c) —1. (d) 0.

8. The value of $\int_{|z|=1} \frac{dz}{z-}$ is ______________

(a) $2\pi i$ (b) 0.

(c) (d) 4.

9. The region of convergence of the series :

$1 \quad \frac{z}{1!} \quad \frac{z}{2!} \quad . \quad \frac{z}{n!} \quad \dots$ is ______________

10. What is the sum function of the series $1 \quad z + z2$ ———z^{n-1}

11. For $f \quad) \quad \frac{-4}{z-2}$, $z = 2$ is a ______________

(a) Removable singular point. (b) Pole of order 1.

(c) Pole of order 2. (d) Essential singular point.

Turn over

12. Identify the poles of :

$$\frac{2z}{z^2 -}$$

(12 X ¼ = 3 weightage)

Section B

Answer all **nine** *questions.*

13. Show that $f'(z)$ does not exist at any point for $f(z) = 2x + ixy$

14. If $f(z)$ and $\overline{f(z)}$ are both analytic in a domain **D,** prove that $f(z)$ is a constant throughout **D.**

15. Show that :

$$\text{Log } (1 - \quad = \frac{1}{2} \text{In } 2 - \frac{}{4}$$

16. Show that :

$$|\sinh z| = \sinh x + \sin y.$$

17. State Cauchy's integral formula.

18. Evaluate :

$$\int \frac{dz}{z-1}, \text{ where C is } |z - 1$$

19. Show that when $z \neq 0$

$$\frac{}{z^2} = \frac{1}{z} \quad \frac{1}{z} \quad \frac{1}{2!} \quad \frac{z}{3!} \quad ..$$

20. State Cauchy's residue theorem.

21. For the function $f(z) = \dfrac{1 - \cosh z}{z^3}$ determine the order of the pole at $z = 0$ and the corresponding residue.

(9 x 1 = 9 weightage)

Section C

Answer any **five** *questions.*

22. Show that $u(x, y) = 2x - x^3 + 3xy$ is harmonic and find a harmonic conjugate $v(x, y)$ of u.

23. If $f(z) = u(x, y) + iV(x, y)$ is analytic in a domain **D,** prove that u and V are harmonic in **D.**

24. Find the general solution of the equation :

$$\cosh z = \frac{1}{2}$$

25. Evaluate $\int_C \frac{z\,dz}{(9 - z^2)(z \quad i)}$, where C is the circle $z \quad = 2$.

26. State and prove Liouville's theorem.

27. State and prove Taylor's theorem.

28. Evaluate $\int_0 \frac{dx}{x^2 + 1}$.

(5 x 2 = 10 weightage)

Section D

Answer any **two** *questions.*

29. State and prpove maximum modulus principle.

30. Give two Laurent series expansions in powers of z for the function :

$$f(z) = \frac{1}{z(1}$$

the regions of validity of expansions.

31 Using residues, evaluate

$$\int_{\infty} \frac{\cos x\,dx}{(x^2 + a^2)(x^2 + b^2} \quad (a > b > 0)$$

(2 x 4 = 8 weightage)