

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours

Maximum : 30 Weightage

Part I

Answer all questions.

1. Define a slack variable.
2. Define a convex set in
3. State True or False :
The union of two convex sets is convex.
4. Define non-degenerate basic solution of the system $AX = B$.
5. What is the optimality criterion for the basic feasible solution of a maximization L.P.P. ?
6. Define artificial variable.
7. The primal has 5 decision variables and 3 constraints. Then its dual has _____ decision variables and _____ constraints.
8. Dual of the dual is the _____
9. Define a loop in a Transportation Problem (TP).
10. State True or False :
A balanced TP always possesses a finite feasible solution and an optimal solution.
11. The number of zeros in a non-degenerate basic feasible solution of a balanced Transportation Problem with 4 sources and 5 destinations is _____
12. The decision variables *in an Assignment problem are
 - (a) 1 only.
 - (b) 0 only.
 - (c) Either 1 or 0.
 - (d) None of these.

(12 x d = 3 weightage)

Turn over

Part II

Answer any nine questions.

13. Write in standard form :

$$\text{Maximize } z = 3x_1 - x_2$$

subject to

$$x_1 - 2x_2 \leq -3$$

$$4x_1 + x_2 < 4$$

$$x_1, x_2 \geq 0.$$

14. Show that $k = \{(x, y) \in \mathbb{R}^2 ; x^2 + y^2 < 9\}$ is convex.

15. Show that every hyperplane in \mathbb{R}^n is convex.

16. Show that a set 'S' is convex in E^n implies that every convex combination of points in S is in S.

17. Show that an optimal solution of

$$\text{Minimize } z = cx$$

subject to $Ax \leq b, x \geq 0$ is also an optimal solution of

$$\text{Maximize } z' = -cx,$$

$$Ax \leq b, x \geq 0.$$

18. Find the dual of :

$$\text{Minimize } z = 4x_1 - x_2$$

subject to

$$x_1 + x_2 \leq 4$$

$$2x_1 - x_2 \geq 3$$

$$x_1 \geq 0$$

x_2 unrestricted.

19. State the Fundamental Theorem of Linear Programming.

20. Find an IBFS by NWCR :

	A	B	C	D	
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
	6	10	12	15	

21. Find an IBFS to the above problem by the matrix minimum method.

22. Test optimality of the Basic Feasible Solution $x_{12} = 30, x_{21} = 10, x_{22} = 10, x_{23} = 30, x_{31} = 10, x_{33} = 10$ for the Transportation Problem given below :

	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	
I	2	1	3	4	30
II	3	2	1	4	50
III	5	2	3	8	20
	20	40	30	10	

23. Give the Mathematical Form of the Assignment Problem.

24. What is a restrictive Assignment Problem and how is it tackled ?

(9 x 1 = 9 weightage)

Part III

Answer any five questions.

25. State and prove a necessary and sufficient condition for a set S in E^n to be convex.

26. Show that the basic feasible solutions of $Ax = b$ are the extreme points.

27. Explain in simple steps the computational procedure of the simplex method.

28. Solve : Maximize $z = x_1 + 5x_2$

subject to

$$x_1 + 10x_2 \leq 20$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

29. Solve Maximize $z = 2x_1 - 3x_2$

subject to

$$-x_1 + x_2 \leq -2$$

$$5x_1 + 4x_2 \leq 46$$

$$7x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

Turn over

30. Find an IBFS by VAM and test for optimality :

A	B	C
10	9	8
10	7	10
11	9	7
12	14	10
10	10	8

31. Solve the following minimization Assignment Problem

	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

32. Prove that in a balanced TP, there are at most $m \times n - 1$ basic variables (m – no: of sources, n – no : of destinations).

(5 x 2 = 10 weights)

Part IV

Answer both questions.

33. Solve the following Transportation Problem to obtain the optimal solution :

	A	B	C	D
I	6	1	9	3
II	11	5	2	8
III	10	12	4	7
85	35	50	45	

34. Solve the following maximization Assignment Problem :

	1	2	3	4	5
A	9	3	4	2	10
B	12	10	8	11	9
C	11	2	9	0	8
D	8	0	10	3	7
7	5	6	2	9	

(2 x 4 = 8 weights)