Reg. No..................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.–CCSS)

Core Course—Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time . Three Hours | Maximum : 30 Weightage

Answer **all** *questions.*

1. Define a harmonic function.

2. Sin $(i\,y)$ = ————

3. Given $f(z) - \dfrac{z\ (z-1)\ (z+1)}{(z+2)^2 . (z+6)}$ .. Write the order of the zero $z = 1$.

4. State Cauchy's residue theorem.

5. Cos $h^2 z$ – sin $h^2 z$ = ————

6. Define isolated singularity.

7. If $e^z = e^{\ y}$ then arg (e^z) ————

8. $\int_c \dfrac{z^2}{z-3}\, dz$ = ———— where c is the circle $|z| = 2$.

9. Prove that $u = x^2 - y^2$ is harmonic.

10. State Liouville's theorem.

11. Verify Cauchy-Riemann equation for the function $f(z) = (3x + y) = i\,(3y \quad x)$.

12. Define Pole.

(12 x 1/4 = 3 weightage)

Answer all **nine** *questions.*

13. Find the harmonic conjugate of $u = x^4 \quad 6x^2y^2 + y^4$.

14. Prove that $f'(z)$ does not exist at any point if $f(z) = z -$

Turn over

15. Prove that $I(z) = {}^{z}$ **does not have a limit when** $z \to 0$.

16. Evaluate $\int \frac{5}{z}\, dz$ **where** c **is** $|z| = 1$.

17. Give an example of removable singularity.

18. Evaluate the residue at the pole $z = 1$ **of** $f(z) = \frac{z+1}{z^2(z-1)}$

19. Determine the order of zero of the function $z(e \quad {}^{-1}$ **at** $z = 0$.

20. Find the principal value of $(-i)^{i}$.

21. Prove that $\exp\left(\quad 4 \quad \right) \qquad \left(\frac{1+i}{\;}\right).$

(9 x 1 = 9 weightage)

Answer any **five** *questions.*

22. Find an analytic function $f(z) = u + iv$ **given** $u = \sin x \cos by + 2 \cos x \sin by + x^2 - {}^{2} + 4xy$.

23. Prove that Log $\quad i) = \frac{1}{2}\,\text{Log } 2 - i\,\frac{\;}{4}$

24. State and prove Cauchy's Integral formula.

25. Using Taylor's series prove that

$$\frac{}{7 - z} = \quad 0 \quad \frac{z}{-} \quad {}^{n}$$

26. Using **Cauchy's residue theorem evaluate** :

$\int \frac{z+1}{z^{=}}\, dz$ **where c is** $|z| = 1$.

27. Prove that differentiable functions are continuous.

28. Let $f(z)$ be an analytic function such that I $f(z)$I A$|z|$ for every z, where A is constant. Prove that $f(z) = a, z$ where a, is a complex constant.

(5 x 2 = 10 weightage)

Answer any two questions.

29. Obtain Laurent series expansion of $\frac{1}{(z-1)(z-2)}$ in

30. Prove that $1(z) = 1 \quad \frac{(\bar{z})^2}{z} \; : z \quad 0$ $\quad z = 0$ is not analytic at z = O. But Cauchy-Riemann equations are satisfied at that point.

31. Evaluate $\int_{0}^{Co} \frac{1}{(x^2+1)^=} dx$

(2 x 4 = 8 weightage)