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# SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.—CCSS)

## Core Course—Mathematics

## MM 6B 09—REAL ANALYSIS

**Time : Three Hours** **Maximum : 30 Weightage**

### Part A

*Answer* **all** *questions.*

1. State Maximum Minimum theorem.
2. Give an example of a uniform continuous function.
3. Define norm of a partition.
4. When is a function $f : [a, b]$ R is Riemann integrable ?
5. Define primitive of *f*.
6. If J = J = $[c, d] \subset [a, b]$ and $\varphi_j(x) = \begin{matrix} 1, x \text{ E } J \\ x \in [a, b] - J \end{matrix}$. Find $_a$ $\varphi_j$ ?
7. Show that $\frac{}{n}$ = 0,V $x$ E R.
8. Evaluate lira $\frac{nx}{1 + (nx)^{2}}$
9. Find the radius of convergence of $\sum x$ .
10. Define improper integral.
11. State True or false. "If f is continuous on [1, 00) and if $\int_1^{00} f(x)\, dx$ converges then $\lim_{x \to \infty} f(x) = o$.
12. What is B (m, $n$) ?

(12 x 3 weightage)

Turn over

## Part B

*Answer all questions.*

13. Define Lipschitz function.

14. State an prove Bolzano's intermediate value theorem.

15. Show that every constant function on [a, $b$] is R [a, $b$].

16. Using First form of fundamental theorem prove that $\int_a f\, x\, dx - \frac{1}{2}\, b^2 \quad a^2)$.

17. If $f_n$ (x) $= x$, $x$ E [0, 1], check whether $f_n(x)$ converges to zero uniformly.

18. If ($f_n$) is a sequence of continuous functions converging uniformly to $f$, prove that $f$ is continuous.

19. Show that $\int^{\infty} — dx$ diverges.

20. Define Gamma function.

21. Show that B (1, n) = 1/n.

(9 x 1 = 9 weightage)

## Part C

*Answer* **any five** *questions.*

22. If $f$ and $g$ are uniform continuous on a subset A of R, then prove that f + $g$ is uniformly continuous on A.

23. Show that $f(x) = xe \quad - 2$ has a root in [0,11

24. If $f$ E R [a, $b$] then prove that $f$ is bounded on $[a, b]$.

25. State and prove Weierstrass M test for the series.

26. If $\int_a (x)|\, dx$ converges absolutely then prove that $\int_a f(x)\, dx$ converges.

27. Show that $\int_0 \frac{dx}{x}$ diverges.

28. Prove the recurrence formula for gamma function.

(5 x 2 = 10 weightage)

## Part D

*Answer any* **two** *questions.*

29. State and prove Squeeze theorem. Also prove that a step function $\varphi : [a, b] \to R$ is in $R\,[a, b]$ .

30. State and prove Cauchy's criterion for uniform convergence.

31. Show that $\int_0 \frac{1}{-x^2}\,dx$ and $\int_0^{\infty} \frac{1}{x^2 + \sqrt{x}}\,dx$ are convergent.

(2 x 4 = 8 weightage)