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# SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics—Core Course

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 30 Weightage

## Section A

*Answer all* **twelve** *questions.*

1. What is the real part of $z^2$ ?
2. The real and imaginary parts of an analytic function are ——— functions.
3. Give any singular point of the function $\frac{2z+1}{z(z^2}$.
4. Choose the correct answer :

   $1\ e21$ = ————

   (a) $e^x$ (b) $e$.

   (d) $e^2$
5. The period of $e^z$ *is* ————

   (a) $2\pi$. (b) 27ti.

   (c) (d) *ni.*
6. Express sin $x$ in terms of ex.
7. What is the parametric form of the unit cicle ?
8. Every bounded entire function is ————
9. The region of convergence of $1 + z + z^2 + \ldots$ *is* ————
10. $\int_{|z|=1} (z2 \quad 1)\, dz =$ ————
11. For the function f $(z) = \frac{\sin}{\quad}, r - 0$ is .

    (a) Pole of order 1. (b) Removable singular point.

    (c) Essential singular point. (d) Pole of order 2.

1. Identify a singular point of $z\ \frac{1}{z^2}$. (12 x ¼ = 3 weightage)

Turn over

## Section B

*Answer all* **nine** *questions.*

13. If $f'(z) = 0$ everywhere in a domain D, prove that $f(z)$ *is* a constant throughout D.

14. Define harmonic function and give example.

15. Show that $\log(-ei) = 1 - \frac{}{2}$

16. Find the principal value of $(-i)^{i}$.

17. State Cauchy-Goursat Theorem.

18. Evaluate $\int \frac{dz}{z - a}$ where C is $|z-a| = R$.

19. State Taylor's theorem

20. Discuss the nature of singularity of $e^{1/z}$ at $z$

21. For the function $(z) = \frac{1-e}{4}$, determine the order of the pole at $z = 0$ and the corresponding residue. **(9 x 1 = 9 weightage)**

## Section C

*Answer any five questions.*

22. Derive the Cauchy-Riemann equations of an analytic function.

23. Show that $u(x, y) = \sinh x \sin y$ is harmonic in a domain and find a harmonic conjugate v x, y) of $u$.

24. Find all roots of the equations.

    (a) $e^z = -2$. (b) $\sin h z = i$.

25. State and prove fundamental theorem of algebra.

26. Evaluate $\int \frac{\exp(2z)}{z} dz$, where C is the circle $|z| = 1$.

27. Obtain the Taylor series expansion of $e^z$ about $z = 1$ and state the region of validity.

28. Evaluate $\int_0 \frac{dx}{(x^2+1)}$. **(5 x 2 = 10 weightage)**

## Section D

*Answer any* **two** *questions.*

**29. State the prove Cauchy's integral formula.**

**30. Give two Laurent series expansions in powers of $z$ for the function $f(z) = \frac{1}{z(1-z)}$ and specify the regions in which the expansions are valid.**

**31. Using residues, evaluate**

$$\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$$

**(2 x 4 = 8 weightage)**