Reg. No.........................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics—Core Course

MM 6B 09—REAL ANALYSIS

Time : **Three Hours** **Maximum : 30** Weightage

I. Objective Type Questions (Answer all *twelve* questions) :

1 State True or False. "Continuous functions are always bounded".

2 Does the function $f(x)$ $x^3 - 4x - 3$ has a root in [2, 3] ?

3 Give an example to a function which is continuous but not uniformly continuous.

4 If I = [0 1], find the norm of the Partion P : = (0 0.2 0.6 0.8 0.9 1).

5 State True *or* False. "An unbounded function cannot be Riemann integrable".

6 Consider the Signum function on [-10, 10]. Then the value of $\int_{-10}^{+10} \mathrm{Sgn}(x)\,dx$ is ———

7 Value of $\lim(x^n)$ for x E (-1, 1) is ______

8 State Weierstrass–M test.

9 Define uniform norm of a bounded function 4) on A c R and 4) : A → R.

10 Give an example of an improper integral.

11 Examine the convergence of $\int \frac{1}{x^2} dx$.

12 Write the relation between Beta and Gamma functions.

(12 x ¼ = 3 weightage)

II. Short Answer Type Questions. (Answer all *nine* questions) :

13 Define a bounded function. Also give an example to a function which is not bounded.

14 Find the absolute maximum and absolute minimum of $g(x) = x^2$ in A : = [-1, 1].

15 State intermediate value theorem.

16 Show that a constant function on $[a\ b]$ is Riemann integrable on [a

17 State fundamental theorem of calculus.

18 Show that $g_n(X) = x^{n}$ for $x \in [0, 1]$; $n \in$ N is not uniformly convergent.

Turn over

19 Test for uniform convergence

$fn(\ ) = \frac{2}{1+7:2x}$ for all real x.

20 Examine the convergence of $1 \frac{dx}{x}$

21 Show that 13 (m, n) m).

(9 x 1 = 9 weightage)

III. Short Essay Questions. (Answer any *five* from seven questions)

22 Let I : [a, b]. Let f : I → R be a continuous function, prove that f is bounded on I.

23 Let I be an interval and f : I → R be continuous on I. Prove that f(I) is an interval in R.

24 If f E R b], prove that, the value of the y . is uniquely determined.

25 Prove that every continuous function is integrable.

26 Prove that A sequence of functions {f} which L bounded on A ⊂ R converges uniformly

to f $\| f_n \quad f \ _A \quad 0.$

27 Show that $\int_{\pi}^{7} \frac{\sin}{} dx$ converges conditionally

28 Prove that $\beta\left(m, \frac{1}{2}\right) = 22m\text{-}1$ m, m).

(5. x 2 = 10 weightage)

IV. Essay Questions (Answer any *two* from three questions) :

29 Let I = [a, b] and f I be continuous if $f(a) < 0 < f(b)$ or if $f(a) > 0 > f(b)$ prove that there exists c in (a, b) with $f(c) = 0$.

30 (i) If f and g are in R [a, b], prove that $f + g$ *is* also in R [a, b].

(ii) State Cauchy 'criterion for integrability of a function and use if to show that the Dirichle

function f 1 when x is rational, 0 when x is irrational is not Riemann integrable in [0 1].

31 (i) State and prove Weierstrass M-test.

(ii) Prove that $\Gamma n = (n \qquad - L$

(2 x 4= 8 weightage)