Reg. No..................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics—Core Course

MM 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

Time : Three Hours | Maximum : 30 Weightage

I. Answer all *twelve* questions.

1. A system of m homogeneous linear equations AX = 0 is unknown has only trivial solution if:

   (a) $m\ n$ (b) $m\ n$

   (c) rank (A) = in . ~~(d) rank (A) = n.~~

2. If the number of variables in a non—homogeneous system AX = B is n then the system possesses a unique solution if :

   p [A, 13]= $\rho(A)$

3. If A is a matrix of order *in* x *ii* and R is a non—singular matrix of order m then $\rho(RA) =$ ________

4. If $n > 2$ then (1)(n) is an ~~integer.~~

5. If a, ***b***, *c* are integers and *g.c.d.* (*a, b, c*) = 1 then *gcd* (*a, b*) = ________________

6. If N is a positive integer then $\sigma(n) =$ ________ $n—1$ $n=1$

7. The value of $\sum_{n=1}^{6} T(n)$ is ______

8. $a'' \ a \pmod{p}$ for any integer a if p is a

9. The linear congruence $ax\ b \pmod{n}$ has a unique solution modulo n if ***g.c.d.*** (*a, n*) = ________

10. If $ca \quad cb \pmod{n}$ and ***g.c.d*** (*c, n*) = 1 then $a \equiv$ ________ (mod n).

11. When two integers a and *b* are said to be relatively prime.

12. Find the k_m (143, 227).

(12 x $\frac{1}{4}$ = 3 weightage)

Turn over

II. Short answer type questions. (Answer all *nine* questions)

13. Prove that *if* ***gcd*** *(a, b)* = *d* then ***g.c.d.*** $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

14. Find the remainder when $1! + 2! + 3! + \ldots\ldots + 99! + 100!$ is divided by 12.

15. State Chinese Remainder theorem.

16. Find $\sigma(180)$.

17. Find $\phi(16)$.

18. Show that no skew—symmetric matrix can be of rank 1.

19. **If A** = $\begin{vmatrix} 2 & 0 \\ 0 & 8 \end{vmatrix}$ then find $\rho(A)$

20. Define Null space of a matrix.

21. Examine whether the system of equations $4x + 6y = 5, 6x + 9y = 7$ has a solution.

(9 x 1 = 9 **weightage**)

III. Answer any *five* questions from seven :

22. Show that every square is of the form 3m or 3m + 1.

23. Prove that the product of any three consecutive integers is divisible by 6.

24. Prove that $\sqrt{2}$ is irrational.

25. Use the binary exponentiation algorithm to compute 5^{110} (mod 131).

26. Reducing to the normal form find the rank of the matrix $\begin{vmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{vmatrix}$

27. If A is a non—singular matrix prove that the **eigenvalues** of **A** are the reciprocals of the **eigenvalues** of A.

28. Verify **Cayley** Hamilton theorem for the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

(5 x 2 = 10 **weightage**)

IV. Answer *two* questions from three.

29. State and prove Wilson's theorem.

30. Prove that $3^{2n+1} \quad {}^{2n+2}$ is divisible by 7.

31. Find the characteristic roots and the corresponding characteristic vectors of the matrix

$$\begin{vmatrix} -2 & -1 \\ 5 & 4 \end{vmatrix}$$

(2 x 4 = 8 weightage)