D 13791 (Pages : 3) Name..............................

Reg. No..............................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Core Course—Mathematics

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 mark.

1. Find symmetric difference of the sets $A = \{1, 2, 3, 4, 5\}$; $B = \{4, 5, 6, 7\}$.
2. Find $A \times B$ if $A = \{x, y\}$ and $B = \{1, 2, 3\}$.
3. Define a symmetric relation on a set A.
4. Let $f : R \to R$ be defined by $f(x) = 2x - 3$. Find a formula for $f^{-1}(x)$.
5. Represent the relation $R = \{(1, 1), (1, 2)\ (1, 3)\ (3, 4)\}$ on $\{1, 2, 3, 4\}$ using a matrix.
6. Let $f : R \to R$ and $g : R \to R$ defined by $f(x) = x^3$ and $g(x) = 2x + 9$ find $f \circ g(x)$.
7. Give an example of a function which is continuous at every value of x.
8. Find $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - x}$.
9. Discuss the behaviour of $f(x) = \dfrac{1}{x^2}$ near $x = 0$.
10. Write the converse of the statement 'If the home team wins, then it is raining'.
11. Write the truth table for the proposition $p \leftrightarrow q$.
12. What is the truth value of $\exists x\, p(x)$ where $p(x)$ is the statement '$x^2 \geq 16$' and the domain consists of positive integers not exceeding 4.

(12 × 1 = 12 marks)

Turn over

Section B

Answer **all** *questions.*
Each question carries 2 *marks.*

13. Determine the power set $P(A)$ of $A = (a, b, c, d)$.

14. Let $S = (1, 2, 3, ... 9)$. Write a partition of S.

15. Let $R = \{(1, 3)(1, 4)(3, 2)(3, 3)(3, 4)\}$ be a relation on $A = \{1, 2, 3, 4\}$. Find $R \circ R$.

16. Let $f : A \to B$, $g : B \to C$ be two functions prove if f and g are one-to-one, then the composition $g \circ f$ is one-to-one.

17. Define a countable set and give an example.

18. Write down the conditions for a function $f(x)$ is continuous at $x = c$.

19. State the Sandwich theorem.

20. Write the negation of the statement :

    "There is an honest politician".

21. Write De Morgan's laws of propositions.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*
Each question carries 5 *marks.*

22. Show $a \equiv b \pmod 5$ is an equivalence relation on the set of all integers.

23. Let $A = \{1, 2\}$, $B = \{a, b, c\}$, $C = \{c, d\}$. Find $(A \times B) \cap (A \times C)$ and $A \times (B \cap C)$.

24. Let $f : A \to B, g : B \to C$, $h : C \to D$. Prove $(f \circ g) \circ h = f \circ (g \circ h)$.

25. Let $A = \{a, b\}$ and $B = \{1, 2, 3\}$. Find the number of functions (1) from A into B ; (2) from B into A.

26. Prove that $\lim_{x \to 2} f(x) = 4$ if

$$f(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2 \end{cases}.$$

27. At what points the function

$$y = \frac{1}{x - 2} - 3x \text{ is continuous.}$$

28. Write the converse, contrapositive and inverse of the conditional statement "If it snows today, I will ski tomorrow".

29. Construct a tenth table for the compound proposition $(p \vee q) \oplus (p \wedge q)$.

30. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*
Each question carries 10 *marks.*

31. (a) Let $A = \{a, b, c, d\}$. Give a relation on A which is (1) Reflexive ; (2) Symmetric ; (3) Antisymmetric.

(b) Define partial ordering relation on a set S. Show that the relation $\leq$ on the set R of real numbers is a partial ordering.

32. (a) Prove that a function $f : A \rightarrow B$ us invertible iff f is one-to-one and onto.

(b) Define recursively defined functions and give an example.

33. (a) Translate into logical expression using predicates and quantifiers. "Someone in your class has visited Mexico". Domain consists of all students in your class.

(b) Show $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent.

(c) Write the negation of the statement :

'All Americans eat cheeseburgers'.

(2 × 10 = 20 marks)