C 33293 (Pages : 4) Name..............................

Reg. No..............................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Mathematics

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Find the number of elements in the power set of {days of the week}.
2. Find A – B for the sets A = {1, 2, 3, 4} , B = {3, 4, 5, 6, 7}
3. Give an example of relation R on A = {1, 2, 3} which is transitive but $R \cup R^{-1}$ is not transitive.
4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 2x$. Then $(f \circ f)(2)$ is :
5. Find the domain of the real valued function $f(x) = \frac{1}{x-2}$.
6. Define a denumerable set.
7. If the graph of a function is symmetric about the origin, then the function is an ———.
8. The graph of $y = x^2$ is shifted 2 units to the left and 2 units up, write the equation of the new graph.
9. Find $\lim\limits_{x \to 1} \frac{x^2 - 1}{x - 1}$.
10. Write the negation of "This is a boring course".
11. What is the truth value of $\forall x \left(x^2 \geq x\right)$ if the domain consists of all real numbers.
12. State which rule of inference is the basis of the argument "It is below freezing now. Therefore it is either below freezing or raining now".

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. If A = $\{a, b, c, d\}$ and B = $\{y, z\}$. Find A × B and B × A.

14. Let R be a relation on A = {1, 2, 3} defined by R = {(1, 1), (1, 2), (2, 3), (3, 1), (3, 2)}. Find R^c and R^{-1}.

15. Find all partitions of S = {1, 2, 3}.

16. Let S = {–1, 0, 2, 5} find f (S) where $f(x) = \left[\frac{x}{5}\right]$.

17. Find the inverse of the function $f(x) = \frac{2x-3}{5x-7}$.

18. Let the functions f and g be defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$. Find (a) $f \circ g$. ; and (b) $g \circ f$.

19. For the function $f(x) = \begin{cases} 0 & x \le 0 \\ \sin \frac{1}{x} & x > 0 \end{cases}$ find $\lim_{x \to 0} f(x)$ or explain why they do not exist.

20. Evaluate $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

21. If $\lim_{x \to 0} f(x) = 1$ and $\lim_{x \to 0} g(x) = -5$ find $\lim_{x \to 0} \frac{2f(x) - g(x)}{(f(x) + 7)^{2/3}}$.

22. Determine whether these biconditions are true *or* false :
    (a) 2 + 2 = 4 if and only if 1 + 1 = 2.
    (b) 0 > 1 if and only if 2 > 1.

23. Compare the terms Tautology and Contradiction.

24. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Let R_1 and R_2 be relations on a set A represented by the matrices :

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Find the matrices representing :

(a) $R_1 \cup R_2$. (ii) $R_1 \cap R_2$.

26. Suppose $\mathscr{C}$ is a collection of relations S on a set A and let T be the intersection of relations S, that is $T = \bigcap \{S \mid S \in \mathscr{C}\}$. Prove that if S is transitive, then T is transitive.

27. Show that $P \times P$ is denumerable, where P is the set of all positive integers.

28. Let R be the relation on P defined by the equation $x + 3y = 12$.

(a) Write R as a set of ordered pairs.

(b) Find (i) Domain of R ; (ii) Range of R ; and (c) R^{-1}.

29. Find the continuous extension to $x = 2$ of the function $f(x) = \dfrac{x^2 + x - 6}{x^2 - 4}$.

30. Show that if $\lim\limits_{x \to c} |f(x)| = 0$ then $\lim\limits_{x \to c} |f(x)| = 0$.

31. Show that $\forall x \, (P(x) \wedge Q(x))$ and $\forall x \, P(x) \wedge \forall x \, Q(x)$ are logically equivalent.

32. Show that the hypothesis $(p \wedge q) \vee r$ and $r \to s$ imply the conclusion $p \vee s$.

33. Prove that "If n is an integer and n^2 is odd then n is odd".

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Consider the following five relations on the set A = {1, 2, 3, 4} :

$R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$; $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$;

$R_3 = \{(1, 3), (2, 1)\}$; $R_4 = \varnothing$, empty relation ; $R_5 = A \times A$.

Determine which of the relations are :

(a) Reflexive.

(b) Symmetric.

(c) Antisymmetric.

(d) Transitive.

35. Let $f(x) = \begin{cases} 3 - x & x < 2 \\ \dfrac{x}{2} + 1 & x > 2 \end{cases}$

(a) Find $\lim\limits_{x \to 2^+} f(x)$ and $\lim\limits_{x \to 2^-} f(x)$.

(b) Does $\lim\limits_{x \to 2} f(x)$ exist ? If so what is it ? If not, why not ?

(c) Find $\lim\limits_{x \to 4^-} f(x)$ and $\lim\limits_{x \to 4} f(x)$.

(d) Does $\lim\limits_{x \to 4} f(x)$ exist ? If so what is it ? If not, why not ?

36. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

(2 × 10 = 20 marks)