Reg. No................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(U.G.–CCSS)

Mathematics—Core Course

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer all **twelve** *questions.*

1. Let f and g are functions on R to R defined by $f(x) = 2x$ and $g(x) = 3x^2 - 1$ then $f \circ g$ is ______
2. State True *or* False : "The set of all integers is countable".
3. Give an example of a set which is bounded above but not bounded below.
4. The **supremum** of the set $S = \frac{1}{\;}; n \in N$ is ______
5. Give an example of a monotonic increasing sequence.
6. Give an example of a Cauchy sequence.
7. State True *or* False : "The empty set ϕ is open and closed".
8. n G= $_{n=1}$ is ______
9. The interior of the set N is ______
10. If z is real, then the relation between z and z is ______
11. $\arg(z_1, z_2) =$ ______
12. " $|z| < 1$ is bounded region. State True *or* False.

(12 x ¼ = 3 weightage)

Part B

Answer all **nine** *questions.*

13. Show that f defined by $f(x \quad \frac{2x}{x-1}$ in $A = \{x \in R \;\; x$ s infective.

Turn over

14. State the principle of strong induction.

15. Prove that the greatest member of a set if it exists is the supremum of the set.

16. Prove that $|a+b| \leq |a| + |b|$.

17. Define convergence of a sequence.

18. Use the definition of the limit of a sequence to prove that $\lim\left(\frac{1}{n}\right) = 0$

19. Prove that every convergent sequence is bounded.

20. Show that subsequence of a convergent sequence is convergent.

21. Show that $\operatorname{Re}(iz) = -\operatorname{I}mz$.

X 1 = 9 weightage)

Part C

Answer any **five** *questions.*

22. Using mathematical induction, prove that $n^3 + 5_n$ is divisible by 6.

23. Prove that the set Q of rational numbers is countable.

24. If M and N are neighbourhoods of a point x, then show that M n N is also a neighbourhood of x.

25. If A and B are non-empty subsets of R such that $a < b$ for all $a \in$ A and b E B, then prove that sup A $\leq$ inf B

26. State and prove squeeze theorem.

27. Define Cauchy sequence. Also prove that every convergent sequence of reals is a Cauchy sequence.

28. Define a connected set in complex plane. Give an example.

(5 x 2 = 10 weightage)

Part D

Answer any **two** *questions.*

29. (a) If $x > -1$, then prove that $(1 +$ $1 + nx$ for all n E N.

    (b) State and prove Nested intervals property.

30. State and prove Bolzano–Weirstrass theorem (second proof).

31. Prove that union of an arbitrary family of open sets is open. What about arbitrary intersection of open sets ? Justify your answer.

(2 x 4 = 8 weightage)