FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(U.G.–CCSS)

Core Course—Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours   Maximum 30 Weightage

Section A

Answer **all** *questions.*
Each question carries $^{1/4}$ *weightage.*

1. **What is the order of** $(y''')^2 + (y'')^7 + y = \sin t$?

2. **Give the general form of a separable equation.**

3. **Test for exactness** : $(2x + 4y)\, dx + (2x - 2y)\, dy = 0$.

4. **Solve** $y'' - \mathbf{y}\ \mathbf{0}$.

5. **State Abel's theorem.**

6. **Find the** Wronskian **of** $y_1 = e^{2t}, y_2 = e^{-3t}$

7. **What is** $L\{e^{-}\}$?

8. **State true** *or* **false** : The Laplace transform is a linear operator.

9. **Find** $L\{\sin 3t\}$

10. **What is the fundamental period of sin 7** *t?*

11. **What is the heat conduction equation** ?

12. **Is the function** $f(x) = x|x|$ **even, odd or neither** ?

(12 x 1/4 = **3** weightage)

Section B

*Answer **all** questions.*
Each question carries 1 weightage.

13. Verify that $y = 3t + t^2$ is a solution of $ty' - y = t^2$.

14. State the existence and uniqueness theorem for first order initial value problems.

15. Verify whether 'y' is an integrating factor of $ydx - 2xdy = 0$.

16. Solve $2y'' - 5y' + 3y = 2e^{-t}$.

17. Solve $y'' + a^2 y = 0$.

18. Find $L\{\sinh 7t\}$

19. Show that convolution is commutative.

20. Show that the sum of two even functions is even

21. Graph the square wave function.

(9 x 1 = 9 weightage)

Section C

*Answer any **five** questions.*
Each question carries 2 weightage.

22. Find an integrating factor and solve :

$$(2x^2 y)\, dx + \left(x^2 y - x\right) dy = 0$$

23. Solve the initial value problem :

$$(y + 2)\, dx + y(x + 4)\, dy \quad ; y(-3) = -1.$$

24. Solve the initial value problem :

$$y'' - 2y' + y = te^{+} , y(0) = 1, y'(0) = 1.$$

25. Solve $y'' + y = \sin t \sin 2t \cdot$

26. Find $L^{-1}\{2/(s^2 + 3s - 4)\}$.

27. Find $\{t^2 e^{at}$

28. Find the Fourier sine series of $(x) = \begin{cases} x, 0 \le x < 1, f \text{ is of} \\ 1, 1 \le x < 2, \text{ period } 4 \cdot \end{cases}$

(5 x 2 = 10 weightage

Section D

Answer any two questions.
Each question carries **4** *weightage.*

29. Solve by the method of variation of parameters :

$$y = \tan t,\ 0 < t < \frac{\pi}{2}$$

30. (0 Using convolution, find $L^{-1} \left[\dfrac{1}{s^2 (s+2)} \right]$

   (ii) Using Laplace transforms, solve $y'' - y' - 6y = 0$
   $$y(0) = 1, y'(0) =$$

31. Find the Fourier series expansion of :

$$f(x) = \begin{cases} -x, -2 & < 0, f(x+ = (x) \\ x, 0 \le x < 2 \end{cases}$$

   Deduce that $\dfrac{\pi^2}{8} = 1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dots .$