Reg. No. ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(UG–CCSS)

Core Course—Mathematics

MM 5B 06—ABSTRACT ALGEBRA

Time : Three Hours | Maximum : 30 Weightage

Part A

Answer **all** *questions.*

1. Is the usual multiplication a binary operation on the set H = $^2 / n \in$

2. Find the sum of 21 and 34 modulo 45.

3. A group homomorphism is one to one if and only if Kerϕ = ___________

4. Describe Klein 4-group.

5. Find the quotient and remainder when —38 is divided by 7.

6. Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$

7. Define orbits of a permutation a of a set A.

8. Find the index of the subgroup H = (0, 31 in the group z_6.

9. Determine whether the map 4 : (R, +) $\rightarrow$ (Z, +) is given by 4) (x) = greatest integer less than or equal to x is a homomorphism.

10. Give an example of a ring with unit element.

11. Define basis of a vector space.

12. Express (3, – 2, 5) as a linear combination of (2, 0, 0), (0, 2, 0) and (0, 0, 2).

(12 x ¼ = 3 weightage)

Part B

Answer **all** *questions.*

13. Find the sum of 23 and 31 modulo 45.

14. Show that the binary structures (Q, +) and (Z_1 +) under usual addition are not isomorphic.

Turn over

15. Show that set of all real numbers other than 1 *fdm* a group under the operation $a * b = a + b - ab$.

16. Find the quotient and remainder when -38 is divided by 9 according to division algorithm,.

17. Express the permutation a = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ & 4 & 2 & 6 & 1 & 3 \end{pmatrix}$ as a product of transpositions.

18. Find the cyclic subgroup generated by 8 in the group Z_{12}.

19. Prove that every group of prime order is cyclic.

20. Find the units in the ring Z_5.

21. If U and W be two subspaces of a vector space V, then prove that U $\cap$ V is also a subspace of V.

(9 x 1 = 9 **weightage**)

Part C

Answer any **five** *questions.*

22. Let G be a group then prove the following :

(a) a^{-} = a for all a E G.

(b) $(a * b)^{-} = b^{-} * a^{-1}$ for all a, b E G.

23. Prove that a subset H of a group G is a subgroup of G if and only if the following conditions are satisfied

"(a) H is closed under the operation in G.

(b) The identity e of G is in H.

(c) For all a E **H** it is true that a^{-1} E **H.**

24. Show that subgroup of a cyclic group is cyclic.

25. Every permutation a of a finite set is a product disjoint cycles. Prove this theorem.

26. Find all the subgroups of Z_{18} and draw the lattice diagram.

27. If p is prime, show that **Zp** has no divisors of 0.

28. Find K such that S = {(2, -1,3), (3, 4, -1), (K, 2, 1)} is **L.I.**

(5 x 2 = 10 **weightage**)

Part D

Answer any **two** *questions.*

29. Prove that every group is isomorphic to a group of permutations.

30. Let ϕ be a homomorphism of a group G into a group G' then prove the following

    (a) If $a \in G$, then (I) $(a^{-1}) = \phi(a)^{-}$.

    (b) **If H** is a subgroup of G, then ϕ [H] is a subgroup of G'.

31. **Let S_1 = {(1, 2, 3), (0, 1, 2), (3, 2, 1)} and S_2** = {(1, – 2, 3), **(-1, 1, – 2), (1, – 3,** 4)} of V_3 determine the basis and dimension of $[s_1] + [s_2]$.

(2 x 4 = 8 weightage)