

**FIFTH SEMESTER B.Sc. DEGREE (U.G.—CCSS) EXAMINATION  
NOVEMBER 2014**

(SDE)

Mathematics

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Two Hours and Forty-five Minutes

Maximum : 27 Weightage

**Part A**

*Answer all the **nine** questions.*

1. Define direct image and inverse image of a set under a function with example.
2. Check whether the function  $f : A \rightarrow \mathbb{R}$ , where  $A = \{x \in \mathbb{R} / x \neq 1\}$  defined as  $f(x) = \frac{1}{x} - 1$  is an injective map
3. Determine the set A of real numbers x such that  $2x + 3 < 6$
4. State the supremum properly and infimum property of  $\mathbb{R}$ .
5. Show that a sequence of real numbers can have atmost one limit.
6. Find  $\lim_{n \rightarrow +\infty} \frac{2n}{n^2}$
7. If  $x = \lim_{n \rightarrow \infty} x_n$  then prove that  $|x| = \lim_{n \rightarrow \infty} |x_n|$
8. Show that : (a)  $\left| e^{iQ} \right| = 1$  ; (b)  $\left[ e^{i\theta} \right] = \cos \theta + i \sin \theta$
9. Prove that z is real if and only if  $z = \bar{z}$ .

(9 x 1 = 9 weightage)

**Part B**

*Answer any **five** questions.*

10. State and prove Bernoulli's inequality.
11. Prove that  $|a + b| \leq |a| + |b|$  for any two real numbers a and b.

12. Prove that  $\sup (a + S) = a + \sup S$  for any non-empty subset  $S$  of  $\mathbb{R}$  that is bounded above and  $a \in \mathbb{R}$ .
13. If the sequences  $(x_n)$  and  $(y_n)$  converge to  $x$  and  $y$  respectively. Show that  $(x_n + y_n)$  and  $(cx_n)$ ,  $c \in \mathbb{R}$  converges to  $x + y$  and  $cx$  respectively.
14. State and prove squeeze theorem for sequences of real numbers.
15. Give an example of a bounded sequence that is not Cauchy.
16. Find the principal argument  $\text{Arg}(z)$  when  $z = \sqrt{z} - i$ .
17. Find all values of  $(-8i)^{1/3}$ .

(5 x 2 = 10 weightage)

### Part C

*Answer any two questions.*

18. Prove the existence of a positive real number  $x$  such that  $x^2 = 2$ .
19. Show that  $(b^n)$  converges if and only if  $0 < b < 1$ .
20. Find all the  $n$ th roots of unity.

(2 x 4 = 8 weightage)