

**FIFTH SEMESTER B.Sc. DEGREE (U.G.—CCSS) EXAMINATION
NOVEMBER 2014**

(SDE)

Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Two Hours and Forty-Five Minutes

Maximum : 27 Weightage

Part B

Section A

Answer all questions.

Each question carries weightage 1.

1. Find the order and degree of $y = x \frac{d^2 y}{dx^2} + \frac{dy}{dx} x$
2. Is $x = a \sin (nt + \phi)$ a solution of $\frac{d^2 x}{dt^2} + n^2 x = 0$? Why ?
3. Is $ydx - xdy + (x^2 + y^2) dx = 0$ exact
4. State the existence and uniqueness theorem for solutions of a first order differential equation.
5. Solve $(ex + 1)y dy = ex (y + 1) dx$.
6. Solve $\frac{d^2 y}{dx^2} + \frac{2}{x} \frac{dy}{dx} = 0$.
7. Give the standard form of a homogenous differential equation of second order.
8. Solve $\frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} + 9y = 0$.
9. Find the complex conjugate and modulus of $z = -1 - \sqrt{3} i$.

(9 × 1 = 9 weightage)

Section B

*Answer any **five** questions.*

Each question carries weightage 2.

10. Test for exactness and solve if possible :

$$(1 + xy) ydx + (1 - xy) xdy = 0.$$

Turn over

11. Find the differential equation of the family of circles with their centres at the origin.
12. Define Wronskian of two functions and use it to check whether $y = xe^x$ and $y_2 = xe^{-x}$ are linearly independent.
13. Solve $\frac{d^2y}{dx^2} + 36y = e^{ix}$.
14. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2x + x^2$.
15. Express in modulus amplitude form :

$$\frac{1}{(2+i)^2} \cdot \frac{1}{(2-i)}$$
16. If z_1 and z_2 are any two complex numbers, find the modulus and amplitude of z_1, z_2 and $\frac{z_1}{z_2}$ and represent them geometrically.

(5 x 2 = 10 weightage)

Section C

III. Answer any two questions. Each question carries weightage 4.

17 Find the solution by the method of variation of parameters :

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^{-x}$$

18 Solve the following :

$$(a) \frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = \sin 4x \sin x.$$

$$(b) \frac{d^2y}{dx^2} + 4y = \cos 2x.$$

19 (a) If $a + ib = \frac{1}{a+ib}$, prove that $(a^2 - b^2)(a^2 + b^2) = 1$

(b) Show that the sum of two complex numbers is again a complex number.

(c) If z is a complex number and \bar{z} its conjugate, find the real and imaginary parts of z in terms of z and \bar{z}

(d) Determine the region in the z plane represented by

$$(i) 1 < |z + 2i| \leq 3. (ii) \frac{\pi}{6} \leq \arg z \leq \frac{\pi}{3}.$$

(2 x 4 = 8 weightage)