Reg. No. ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2011

(CCSS)

Core Course—Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours | Maximum Weight 30

I. Answer *all* questions

1. Does the differential equation $\frac{dy}{dt} = y$ have a solution passing through the point (1, 0) ?

2. Is the differential equation $\frac{dy}{dt} + ty = 0$ linear or non linear ?

3. Give an integrating factor of the equation : $ydx - xdy = 0$.

4. The differential equation M (x, y) + N (x, *y) y' =0 is* exact if and only if

5. Give the general solution of $\frac{y}{dt} + b\frac{dv}{dt} + cy = 0$ whose characteristic equation has a root

   $+$

6. Write a differential equation whose general solution is $+ c_2te^t$

7. Are the functions cos *t* and sin *t* linearly independent ?

8. The Laplace Transform of the function $t^2 \quad e^{2t}$ is

9. If $h(t) = u_1(t) - u_2(t)$ where u_1 and u_2 are unit step functions, then $h(t)$– ————

   if 1 $t < 2$.

10. If $F(s) =$ $(f$ *On* exists for $s > a$ 0 and c is a constant, then $\mathcal{L}(e^c f(t) =$ ————

    $s > a + c.$

11. The fundamental period of the function $\cos(2\pi x)$ is ————

12. State whether the function $f(x) = x.\cos$ x is even or odd.

(12 x – = 3)

Turn over

II. Answer *all* questions.

13. Solve the initial value problem : $y =$; $y(0) = 0$ and give an interval in which the solution exists.

14. Prove .that $t(x, y) = x$ is an integrating factor of the differential equation $(3xy + y^2) + (x^2 + xy)y = 0$.

15. Obtain the general solution of the equation $16y'' - 8y' + 145y = 0$.

16. Prove that t'' and t^{1} form a fundamental set of solutions of the equation $2t\ y'' + 31y' - y = 0$.

1.7. Prove that $f * g = g * f$, where * denotes the convolution product.

18. Find the Laplace Transform of the unit step function $u_c(t)$.

19. Find the Inverse Laplace Transform of the function $\frac{s^2}{s^4 \quad 1}$

20. If f and g are periodic functions with same period T, show that any linear combination off and g is also T-periodic.

21. Let $f(x) = x$ where 0 x 1. Find the 2-periodic even extension off.

(9 x 1= 9)

III. Answer any *five* questions.

22. Solve the initial value problem $2dx + ye^{-x}dy = 0$; $y(0) = 0$.

23. Solve the initial value problem $y'' y' + 0.25y = 0$; $y(0) = 2$ and $y'(0) = 3$

24. If y_1 and y_2 are solutions of $y'' + p(t)y' + q(t)y = 0$ where $p(t)$ and $q(t)$ are continuous functions of t, prove that for any two constants c_1 and c_2 the linear combination $c_1y_1 + c_2y_2$ is also a solution.

25. Find the Laplace Transform of the function $e^{at} \cos bt$.

26. Prove that the Laplace Transform is a linear operator.

27. Prove that $u(x, t) =$ .sin x is a solution of the equation : $a\ u_{xx} = u_t$.

28. Solve : X' $3X - 2Y$; Y' $2X - 2Y$; $X(0) = 3$ and $Y(0) = \frac{1}{2}$

(5 × 2 = 10)

N. Answer any *two* questions.

29. Find the general solution of $y'' \; 3\,y' - 4y = 3e^{2'} + 2 \sin t$.

30. Using Laplace Transforms, solve : $y'' + y = \sin 21$; $y(0) = 2$ and $y'(0) = 1$.

31. Let $1(x) = 1 \quad x^2$ if - $\leq x$ 5_ 1 and $f(x + 2) = f(x)$. Then

(i) Sketch the graph of the function f and state whether the function is even or odd.

(ii) Find the fourier series of f.

(iii) Deduce that : $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \quad = \quad {}_{1}\, n^{2}$

(2 x 4 = 8)